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THE AUTOMORPHISM GROUPS OF DOMAINS AND THE GREENE–KRANTZ CONJECTURE

Steven G. Krantz

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Abstract. We consider the subject of the automorphism groups of domains in complex space. In particular, we describe and discuss the noted Greene–Krantz conjecture.

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1. INTRODUCTION

It is a standard and well-known fact, due to Poincaré and now over 100 years old, that the unit ball in $\mathbb{C}^n$ and the unit polydisc in $\mathbb{C}^n$ are *not* biholomorphic equivalent. These two domains are obviously the two most natural generalizations of the planar unit disc to higher dimensions. The natural conclusion to draw from this example is that there is no Riemann mapping theorem in several complex variables.

As a possible ersatz for the above described lacuna, it has become natural to study the automorphism groups of domains. Here a *domain* Ω is a connected, open set. An *automorphism* of Ω is a biholomorphic self-map of the domain. The automorphisms form a group under the binary operation of composition of mappings. We denote the automorphism group by $\operatorname{Aut}\Omega$. It is known (see [14, Chapter 5]) that (at least in the case when Ω is bounded) the automorphism group is a Lie group. The topology is the compact-open topology (equivalently, uniform convergence on compact sets). It is usually (in the case when Ω is bounded) a real Lie group but not a complex Lie group.

It is of particular interest to study domains with non-compact automorphism group.

If $P \in \partial\Omega$ then we say that P is a *boundary orbit accumulation point* if there is an $X \in \Omega$ and a sequence of automorphisms $\{\varphi_j\}$ such that $\varphi_j(X) \to P$ as $j \to \infty$.

It is known that a bounded domain with non-compact automorphism group will have a boundary orbit accumulation point (again see [14, Chapter 1]).

Let us recall that, if $\Omega \subseteq \mathbb{C}^n$ is a smoothly bounded domain given by

$$\Omega = \{z \in \mathbb{C}^n : \rho(z) < 0\},$$

with $\nabla\rho \neq 0$ on $\partial\Omega$ and if $P \in \partial\Omega$, then we say that P is *Levi pseudoconvex* if

$$\sum_{j,k=1}^{n} \frac{\partial^2}{\partial z_j \partial \overline{z}_k}(P) w_j \overline{w}_k \geq 0 \tag{1.1}$$

for all complex tangent vectors w which satisfy

$$\sum_{j=1}^{n} \frac{\partial \rho}{\partial z_j} w_j = 0.$$

We say that P is *strongly Levi pseudoconvex* if the Levi form (1.1) is positive for all $w \neq 0$. It is useful to note that a strongly pseudoconvex point is locally biholomorphically equivalent to a strongly convex point (see [21, Chapter 3]).

One of the important classic results in the subject is the famous theorem of Bun Wong [28] and Rosay [25]:

Theorem 1.1. *Let $\Omega \subseteq \mathbb{C}^n$ be a bounded domain. Let $P \in \partial\Omega$ be a boundary orbit accumulation point. If the boundary is C^2 smooth near P and if P is strongly pseudoconvex, then Ω is biholomorphic to the unit ball in $\mathbb{C}^n$.*

Sketch of proof. Since P is strongly pseudoconvex, it is a peak point for the algebra of functions continuous on the closure and holomorphic on the interior (see [10] for details). As a result, if φ_j are as in the definition of boundary orbit accumulation point, then $\{\varphi_j\}$ converges uniformly on compact subsets of Ω to P. Let K be a large compact subset of Ω. So we have a map from K to a small neighborhood of P and this is followed by a map into the ball. (This is actually a special instance of ideas that we saw earlier.)

Now it is also known that a boundary neighborhood of a strongly pseudoconvex point agrees, up to fourth order, with a piece of the boundary of the unit ball B (see [8]). And now we can apply an automorphism of the ball to map this last image to a large compact set in B.

In sum, we have constructed a holomorphic map from a large compact set in Ω to a large compact set in B. In fact, we have a sequence of such maps. It can be shown that these in fact converge to a biholomorphic map of Ω to B. □

Again, the reference [14, Chapter 1] is a good source for background and proofs of this result. One can think of this theorem as a substitute for the Riemann mapping theorem, although it is many respects weaker than Riemann's original result.

2. THE SCALING METHOD

Here we shall say a few words about what the scaling method is. The scaling method can be used to prove the Bun Wong/Rosay theorem. It is also used in many of the results related to the Greene–Krantz conjecture (see the discussion in Section 3). Here we give a very brief precis of what this method is all about. For a fuller treatment, see for instance [14], the last two chapters.

Given a domain Ω in $\mathbb{C}^n$ with an interior point q and a sequence f_j of automorphisms of Ω such that $\lim_{j\to\infty} f_j(q) = p$, for some boundary point $p \in \partial\Omega$, one follows the steps below:

Step 1. Localization. Translate Ω if necessary so that p becomes the origin. Establish that, for any compact subset K of Ω, the sequence of sets $f_j(K)$ shrinks successively to the boundary point $p = 0$.

Step 2. Centering. Adjust the domain Ω by a sequence of complex affine maps, say Ψ_j, so that the new point sequence $\Psi_j(q)$ behave as if it converges non-tangentially to the boundary of the limit domain.

Step 3. Stretching. Find a divergent sequence of complex linear maps, say L_j, so that $\sigma_j = L_j \circ \Psi_j \circ f_j$ converges uniformly on compact subsets of Ω into $\mathbb{C}^n$. (One might say here that we seek a limit map that is nonconstant.)

Step 4. Analysis of the Limit Domain. Since $f_j(\Omega) = \Omega$, it follows that $\sigma_j(\Omega) = L_j \circ \Psi_j(\Omega)$ for every j. Since the maps L_j and Ψ_j are often explicit, take the limit domain $\widehat{\Omega}$ of $\sigma_j(\Omega)$ in the sense of normal convergence of domains.

Step 5. Synthesis. In case all the pieces are put together, it usually follows that σ_j converges to a map $\widehat{\sigma}$ that turns into a biholomorphic mapping from Ω onto $\widehat{\Omega}$.

The localization followed by centering and stretching constitutes the scaling method. The main thrust is that the limit domain becomes much simpler. For example, the limit domain is the Siegel upper-half space (biholomorphic to the ball) in the case when p is a C^2 strongly pseudoconvex boundary point.

The scaling method is essential in many treatments of the Bun Wong/Rosay theorem. In the next section, we will explore some generalizations of this result.

3. SOME GENERALIZATIONS

Certainly some of the most interesting results that generalize Theorem 1.1 are those of Bedford and Pinchuk. We describe some of them now.

Theorem 3.1 ([2]). *Let $\Omega \subseteq \mathbb{C}^2$ be a bounded pseudoconvex domain with real analytic boundary. If Ω has non-compact automorphism group, then Ω is biholomorphic to an egg domain of the form*

$$E_k = \{(z_1, z_2) \in \mathbb{C}^2 : |z_1|^2 + |z_2|^{2k} < 1\}$$

for k a positive integer.

D. Catlin has observed [6] that this last theorem is true also for domains of finite type in $\mathbb{C}^2$. (See the discussion following Lemma 4.2 for this concept.)

Theorem 3.2 ([3]). *Let $\Omega \subseteq \mathbb{C}^{n+1}$ be a bounded pseudoconvex domain of finite type with smooth boundary. Assume that the Levi form has rank at least $n-1$ at each boundary point. Further assume that the automorphism group is non-compact. Then Ω is biholomorphically equivalent to a domain of the form*

$$F_m = \left\{ (w, z_1, z_2, \ldots, z_n) : \sum_{j=1}^{n} |z_j|^2 + |w|^{2m} < 1 \right\}$$

for some positive integer m.

Theorem 3.3 ([4]). *Let $\Omega \subseteq \mathbb{C}^{n+1}$ be convex, smoothly bounded, and of finite type. Assume that Ω has non-compact automorphism group. Then Ω is biholomorphically equivalent to a domain of the form*

$$H_p = \{(w, z_1, z_2, \ldots, z_n) : |w|^2 + p(z, \overline{z}) < 1\},$$

where p is a weighted, homogeneous, real polynomial.

Theorem 3.4 ([5]). *Let $\Omega \subseteq \mathbb{C}^2$ be a bounded domain with real analytic boundary. If Ω has non-compact automorphism group, then Ω is biholomorphic to an egg domain of the form*

$$E_k = \{(z_1, z_2) \in \mathbb{C}^2 : |z_1|^2 + |z_2|^{2k} < 1\}$$

for k a positive integer.

What is noteworthy about this last result is that it is similar to Theorem 3.1 but without the hypothesis of pseudoconvexity.

The primary methodology in all four of these theorems is the method of scaling that has been developed by Frankel [9], Pinchuk [24], and Kim [17]. (Let us just say that these scaling methods differ in the way that the domains or mappings are normalized.) Certainly this has become one of the most powerful and pervasive techniques in the study of automorphism groups.

4. THE GREENE–KRANTZ CONJECTURE

One interesting result in the subject (for which see [14, Chapter 1]) is this:

Theorem 4.1. *Let Ω be a smoothly bounded domain and let $P \in \partial\Omega$ be a boundary orbit accumulation point. Then P must be Levi pseudoconvex.*

There is also a version of this result for domains with non-smooth boundary. It is worth comparing this theorem with the Bun Wong/Rosay theorem. How essential is strong pseudoconvexity in that result?

Proof. Assume the contrary, that $\partial\Omega$ is not pseudoconvex at p_0. Then there exists a compact set K contained in Ω such that the holomorphic hull $\widehat{K}$ of K contains a set of the form $\Omega \cap U$ where U is an open set in $\mathbb{C}^n$ containing p_0.[1)] Recall that the holomorphic hull $\widehat{K}$ of a compact set K is by definition the set

$$\{p \in \Omega : |f(p)| \le \max_K |f|, \forall f : \Omega \to \mathbb{C} \text{ holomorphic}\}.$$

Now choose an $\epsilon > 0$ such that $B^n(x_0, 3\epsilon) \subset \Omega$. Let A_M be the set of $\varphi \in \mathrm{Aut}\,(\Omega)$ such that $\|d\varphi^{-1}|_{\varphi(x_0)}\| \le M$, where $\|\cdot\|$ here represents the usual operator norm. Then we show:

Lemma 4.2. *There exists $\delta > 0$ such that $\varphi(B^n(x_0, \epsilon))$ contains $B^n(\varphi(x_0), \delta)$ for every $\varphi \in A_M$.*

Proof of Lemma 4.2. Since

$$d\varphi^{-1}|_{\varphi(x_0)} = (d\varphi|_{x_0})^{-1},$$

we see that

$$\|(d\varphi|_{x_0})^{-1}\| \le M$$

whenever $\varphi \in A_M$. Consider the map

$$T(z) := (d\varphi|_{x_0})^{-1} \circ \varphi(z), \quad z \in B^n(x_0, \epsilon).$$

The differential at x_0 of this map is equal to the identity. And its second derivatives on $B^n(x_0, \epsilon)$ are bounded (Cauchy estimates on φ) by a constant depending only on M and the bound on $\|(d\varphi|_{x_0})^{-1}\|$ (and Ω and ϵ) but not on $\varphi \in A_M$. Hence, by standard information about the inverse function theorem, $T(B^n(x_0, \epsilon))$ contains a ball of radius $\alpha > 0$ centered at x_0, where α is independent of which φ is chosen from A_M: here α depends only on M (and ϵ and Ω). Thus, the image of the map $\varphi = d\varphi|_{x_0} \circ T$ contains a ball of radius $\delta > 0$ centered at $\varphi(x_0)$, with δ independent of the choice of φ. (The radius δ depends only on M, ϵ and Ω for the following reason: since $d\varphi|_{x_0}$ is a linear transformation with its inverse bounded above in operator norm, no such φ can take a given radius ball to a set not containing a definite radius ball. In fact, it cannot contract anything by more than a factor of $1/M$). Thus, the assertion of the lemma follows. □

Altogether, one obtains that, if $\varphi_j(x_0) \to p_0 \in \partial\Omega$ as $j \to \infty$, then

$$\|d\varphi_j^{-1}|_{\varphi_j(x_0)}\| \to \infty.$$

Let $\psi_j = (\psi_j^1, \ldots, \psi_j^n)$ be the component representation of φ_j^{-1} for a moment. Passing to a subsequence, we may assume that

$$\left| \frac{\partial \psi_j^\ell}{\partial z_m}\Big|_{\varphi_j(z_0)} \right| \to \infty$$

[1)] The usual construction of a compact set in Ω with holomorphic hull running out to a non-pseudoconvex boundary is casually called a “Hartogs tin can” in several complex variables. In case one “Hartogs tin can” does not provide a U of the sort we are after, one can perturb it and take the set K as the union of the perturbations to get the desired situation.

for some $\ell, m \in \{1, \ldots, m\}$. (Otherwise these φ_js would belong to A_M for some $M > 0$, and hence the image of φ_j contains a ball of radius δ, independent of j. A contradiction.) However, this is impossible, because $|\partial\psi_j^\ell/\partial z_m|$ is bounded near p_0 by its absolute value on the compact "Hartogs figure" K and that is bounded by a constant independent of j, by Cauchy estimates. This completes the proof. □

Let us now recall the notion of finite type as first formulated by Kohn [20]. Let $\Omega \subset \mathbb{C}^2$ be smoothly bounded and let $P \in \partial\Omega$. We say that P is of *finite type* m if there is a one-dimensional complex curve γ that has order of contact m with $\partial\Omega$ at P but no complex curve with higher order of contact.

Of course there is a notion of finite type in higher dimensions, but it is rather subtle, and we cannot treat it here. See [7], Chapter 1 and also the discussion in [21].

The ideas in the proof of Theorem 1.1 led to the following conjecture (see [13]):

Conjecture (The Greene–Krantz Conjecture)**.** *Let Ω be a smoothly bounded domain in $\mathbb{C}^n$. Let P be a boundary orbit accumulation point. Then P must be of finite type in the sense of Kohn/D'Angelo/Catlin.*

Although there are some encouraging partial results for this conjecture, the full result is still wide open. We now discuss some of these partial results.

Theorem 4.3 ([18])**.** *Any bounded convex domain in $\mathbb{C}^n$ with a piecewise smooth Levi flat boundary which possesses a noncompact automorphism group is biholomorphic to a product domain.*

This result is proved using a scaling method. It is like an obverse of the Greene–Krantz conjecture.

Theorem 4.4 ([22])**.** *The Greene–Krantz conjecture is true for smoothly bounded convex domains in $\mathbb{C}^2$.*

Interestingly, the proof of this result does *not* use the scaling method. Instead, it uses the idea of subelliptic estimates for the $\overline{\partial}$-Neumann problem. In fact, classical examples of Kohn, Krantz, and others (in dimension 2) exploit finite type in decisive ways.

Theorem 4.5 ([23])**.** *A bounded domain Ω in $\mathbb{C}^2$ with a boundary orbit accumulation point P is biholomorphic to $\{(z,w) \in \mathbb{C}^2 : \mathrm{Im}(w) > k(z,z)\}$, where ρ_k is a homogeneous polynomial with degree k.*

The following two papers also contain interesting partial results about the Greene–Krantz conjecture: [26, 27]. They are only available in preprint form.

5. SEMI-CONTINUITY OF AUTOMORPHISM GROUPS

One of the more interesting and original results in the subject was proved by Greene and Krantz in [11]. Although we cannot prove it in the present survey, we can formulate the result and make some remarks about its meaning.

Let $\mathcal{O}$ be the collection of smoothly bounded, strongly pseudoconvex domains in $\mathbb{C}^n$. If Ω is such a domain then we think of it as the sublevel set of a defining function:

$$\Omega = \{z \in \mathbb{C}^n : \rho(z) < 0\}.$$

It is standard to assume that $\nabla\rho \neq 0$ on $\partial\Omega$ – just to ensure that the boundary has no singularities. Now there is a natural C^∞ topology on the collection of all defining functions. And now we think of that as a topology on the set of all strongly pseudoconvex domains.

Theorem 5.1. *Let Ω_0 be a fixed, smoothly bounded, strongly pseudoconvex domain in $\mathbb{C}^n$. There is a neighborhood $\mathcal{U}$ of Ω_0 in the smooth topology on domains so that if $\Omega \in \mathcal{U}$, then* $\mathrm{Aut}(\Omega)$ *is a subgroup of* $\mathrm{Aut}(\Omega_0)$. *Furthermore, there is a diffeomorphism* (*not a biholomorphism!*) $\Phi : \Omega \to \Omega_0$ *so that the map*

$$\mathrm{Aut}(\Omega) \ni \psi \mapsto \Phi \circ \psi \circ \Phi^{-1} \in \mathrm{Aut}(\Omega_0)$$

explicitly realizes $\mathrm{Aut}(\Omega)$ *as a subgroup of* $\mathrm{Aut}(\Omega_0)$.

The proof of this result is long and complicated, and we cannot treat it in the present context. We merely remark that this theorem actualizes an idea that is intuitively appealing. Namely, a small perturbation of a geometric object can destroy symmetry, but it cannot create symmetry. That is precisely what our semi-continuity theorem says.

This theorem has been generalized to broader classes of domains, and the topology on domains has been weakened considerably (see, for instance [15]). See [14, Chapter 1] for some discussion of these ideas.

6. LARGE ISOTROPY GROUP

We recall the notion of isotropy group. Let $\Omega \subseteq \mathbb{C}^n$ be a domain and $P \in \Omega$. Then the isotropy group I_P of P is the set of automorphisms that fix P. It is clear that I_P is a group – indeed a compact group.

A classical theorem of Bruné, Aumann, Carathéodory *et al.* says the following:

Theorem 6.1. *If Ω is a bounded domain in $\mathbb{C}$ and if, for some $p \in \Omega$, I_p is infinite, then Ω is biholomorphic to the unit disc.*

This result is intuitively clear. For the isotropy group must be a Lie group; since it is infinite, it must be the entire circle. From this it is not difficult to argue that the domain is biholomorphically the disc.

Naturally we would like to know an analogue of this result in higher dimensions. It is clear that infinitude of the isotropy group is not sufficient. For example, the domain

$$E = \{(z_1, z_2) \in \mathbb{C}^2 : |z_1|^2 + |z_2|^4 < 1\}$$

has infinite isotropy group at the origin, but it is *not* biholomorphic to the unit ball. So something more must be hypothesized.

In fact, we have the following result of Greene/Krantz [12].

Theorem 6.2. *Let M be a noncompact complex manifold of complex dimension n. Let $p \in M$. Assume that there is a compact subgroup H of the isotropy group I_p of p with the following property: for any two real tangent vectors η, ξ at p there is an element $h \in H$ such that $dh|_p(\eta) = \lambda\xi$ for some real number λ. Then M is either biholomorphic to the unit ball in $\mathbb{C}^n$ or biholomorphic to $\mathbb{C}^n$.*

The hypothesis of compactness of the subgroup H is essential. An example is provided by $\mathbb{C}^2 \setminus \{(1,0)\}$, $p = (0,0)$. Here the group I_p acts transitively on real directions at p, though no compact subgroup of I_p does – so there is no inconsistency with Theorem 6.2. To see that I_p does act with this transitivity, let v, w be nonzero tangent vectors at $p = (0,0)$. Choose a complex nonsingular linear transformation $A : \mathbb{C}^2 \to \mathbb{C}^2$ with $Av = w$. Of course, A may not induce a map of $\mathbb{C}^2 \setminus \{(1,0)\}$ into itself since $A((1,0))$ may not equal to $(1,0)$. However, a biholomorphic map $\widehat{A}$ from $\mathbb{C}^2 \setminus \{(1,0)\}$ to itself fixing $(0,0)$ and having $d\widehat{A}\big|_{(0,0)} = A$ can be obtained by composing $A : \mathbb{C}^2 \to \mathbb{C}^2$ with suitable "shears" of the forms

$$(z,w) \mapsto (z, w + f(z)) \quad \text{and} \quad (z,w) \mapsto (z + g(w), w),$$

where $f, g : \mathbb{C} \to \mathbb{C}$ are holomorphic functions. In particular, we can require $f(0) = g(0) = 0$ and $df|_0 = dg|_0 = 0$ and still have compositions of such "first order constant" shears acting transitively on $\mathbb{C}^2 \setminus \{(0,0)\}$. Then a composition $F : \mathbb{C}^2 \to \mathbb{C}^2$ of such shears has

$$F((0,0)) = (0,0), \quad dF|_{(0,0)} = \text{identity}, \quad \text{and} \quad F(A((1,0))) = (1,0).$$

The map $\widehat{A}$ defined by

$$(z,w) \mapsto F(A((z,w)))$$

then takes $\mathbb{C}^2 \setminus \{(1,0)\}$ biholomorphically to itself, takes $(0,0)$ to $(0,0)$, and has differential at $(0,0)$ taking v to w.

The theorem of Greene/Krantz was generalized to the compact case by Bland, Duchamp, and Kalka [1]. The geometric hypotheses are the same, but now the conclusion is that the object of study is projective space.

7. CONCLUDING REMARKS

For the past fifty years there has been considerable activity in the study of automorphism groups of smoothly bounded domains in $\mathbb{C}^n$. Certainly the theorem of Bun Wong and Rosay (Theorem 1.1) gave real impetus to these investigations. Those papers inspired the subsequent collaboration of Greene and Krantz, which has produced thirteen papers in the subject. Also, the work of Isaev and Krantz (see for instance [16]) was inspired by these developments. And the collaboration of Bedford and Pinchuk was an outgrowth of this circle of ideas.

The study of automorphism groups is a rich one, involving techniques from complex analysis, Lie theory, differential geometry, partial differential equations, and many other parts of mathematics. Interestingly, the paper [19] uses both the Axiom of Choice and the Continuum Hypothesis.

Many avenues of exploration are still open in this subject area. We hope that this brief exposition will inspire some new workers to join the effort.

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REFERENCES

[1] J. Bland, T. Duchamp, M. Kalka, *A characterization of CP^n by its automorphism group*, [in]: S.G. Krantz (ed.), *Complex Analysis*, Lecture Notes in Mathematics, vol. 1268, Springer, Berlin, Heidelberg, 1987, 60–65.

[2] E. Bedford, S. Pinchuk, *Domains in $\mathbb{C}^2$ with noncompact groups of holomorphic automorphisms*, Mat. Sb. (N.S.) **135** (1988), 147–157 [in Russian].

[3] E. Bedford, S. Pinchuk, *Domains in $\mathbb{C}^{n+1}$ with noncompact automorphism group*, J. Geom. Anal. **1** (1991), 165–191.

[4] E. Bedford, S. Pinchuk, *Convex domains with noncompact groups of automorphisms*, Mat. Sb. **185** (1994), 3–26 [in Russian], translation in Russian Acad. Sci. Sb. Math. **82** (1995), 1–20.

[5] E. Bedford, S. Pinchuk, *Domains in $\mathbb{C}^2$ with noncompact automorphism groups*, Indiana Univ. Math. J. **47** (1998), 199–222.

[6] D. Catlin, *Subelliptic estimates for the $\overline{\partial}$-Neumann problem*, Ann. Math. **126** (1987), 131–192.

[7] J.P. D'Angelo, *Several Complex Variables and the Geometry of Real Hypersurfaces*, Studies in Advanced Mathematics, CRC Press, Boca Raton, FL, 1993.

[8] C. Fefferman, *The Bergman kernel and biholomorphic mappings of pseudoconvex domains*, Invent. Math. **26** (1974), 1–65.

[9] S. Frankel, *Complex geometry of convex domains that cover varieties*, Acta Math. **163** (1989), 109–149.

[10] I. Graham, *Boundary behavior of the Carathéodory and Kobayashi metrics on strongly pseudoconvex domains in $\mathbb{C}^n$ with smooth boundary*, Trans. Amer. Math. Soc. **207** (1975), 219–240.

[11] R.E. Greene, S.G. Krantz, *The automorphism groups of strongly pseudoconvex domains*, Math. Ann. **261** (1982), 425–446.

[12] R.E. Greene, S.G. Krantz, *Characterization of complex manifolds by the isotropy subgroups of their automorphism groups*, Indiana Univ. Math. J. **34** (1985), 865–879.

[13] R.E. Greene, S.G. Krantz, *Invariants of Bergman geometry and the automorphism groups of domains in* $\mathbb{C}^n$, [in:] *Geometrical and algebraical aspects in several complex variables (Cetraro, 1989)*, 107–136, Sem. Conf., 8, EditEl, Rende, 1991.

[14] R.E. Greene, K.T. Kim, S.G. Krantz, *The Geometry of Complex Domains*, Progress in Mathematics, 291, Birkhäuser Boston, Ltd., Boston, MA, 2011.

[15] R.E. Greene, K.T. Kim, S.G. Krantz, A. Seo, *Semicontinuity of automorphism groups of strongly pseudoconvex domains: the low differentiability case*, Pacific J. Math. **262** (2013), 365–395.

[16] A.V. Isaev, S.G. Krantz, *Domains with non-compact automorphism group: a survey*, Adv. Math. **146** (1999), 1–38.

[17] K.-T. Kim, *Complete localization of domains with noncompact automorphism group*, Trans. Amer. Math. Soc. **319** (1990), 139–153.

[18] K.-T. Kim, *Domains in* $\mathbb{C}^n$ *with a piecewise Levi flat boundary which possess a noncompact automorphism group*, Math. Ann. **292** (1992), 575–586.

[19] K.-T. Kim, S.G. Krantz, *Normal families of holomorphic functions and mappings on a Banach space*, Expo. Math. **21** (2003), 193–218.

[20] J.J. Kohn, *Boundary behavior of* $\overline{\partial}$ *on weakly pseudoconvex manifolds of dimension two*, J. Differential Geom. **6** (1972), 523–542.

[21] S.G. Krantz, *Function Theory of Several Complex Variables*, 2nd ed., AMS Chelsea Publishing, Providence, RI, 2001.

[22] S.G. Krantz, *The Greene–Krantz conjecture in dimension two*, Rocky Mountain J. Math. **46** (2016), 1575–1586.

[23] B. Liu, *Analysis of orbit accumulation points and the Greene–Krantz conjecture*, J. Geom. Anal. **27** (2017), 726–745.

[24] S. Pinchuk, *The scaling method and holomorphic mappings*, [in:] *Several Complex Variables and Complex Geometry*, Part 1 (Santa Cruz, CA, 1989), Proc. Sympos. Pure Math., 52, Part 1, Amer. Math. Soc., Providence, RI, 1991.

[25] J.-P. Rosay, *Sur une caractérisation de la boule parmi les domains de* $\mathbb{C}^n$ *par son groupe d'automorphismes*, Ann. Inst. Four. Grenoble **29** (1979), 91–97.

[26] G. Tsai, *A characterization of bounded convex domains in* $\mathbb{C}^n$ *with non-compact automorphism group*, preprint.

[27] N. Van Thu, *Problems about the Greene–Krantz conjecture*, preprint.

[28] B. Wong, *Characterizations of the ball in* $\mathbb{C}^n$ *by its automorphism group*, Invent. Math. **41** (1977), 253–257.

Steven G. Krantz
sk@math.wustl.edu
https://orcid.org/0000-0003-0902-2014

Department of Mathematics
Washington University in St. Louis
St. Louis, Missouri, USA 63130

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