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A GENERAL ELLIPTIC EQUATION WITH INTRINSIC OPERATOR

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Abstract. Existence and bound of a solution is established for a general elliptic equation with intrinsic operator subject to Dirichlet boundary condition. This provides a sufficient condition to the fundamental question if there is a solution belonging to a prescribed ball in the function space. An application deals with an equation involving a convolution product.

Keywords: nonlinear elliptic equation, intrinsic operator, convolution.

Mathematics Subject Classification: 35J92, 47F10.

1. INTRODUCTION AND MAIN RESULT

In this paper we focus on the following quasilinear Dirichlet problem

$$\begin{cases} -\Delta_p u = f(x, Bu, \nabla(Bu)) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega \end{cases} \tag{P}$$

on a bounded domain Ω in $\mathbb{R}^N$ with Lipschitz boundary $\partial\Omega$. For a later use we denote by $|\Omega|$ the Lebesgue measure of Ω. The driving operator in the equation of (P) is the negative p-Laplacian $-\Delta_p : W_0^{1,p}(\Omega) \to W^{-1,p'}(\Omega)$, with $p \in (1, +\infty)$, which is given by

$$\langle -\Delta_p u, v \rangle = \int_\Omega |\nabla u(x)|^{p-2} \nabla u(x) \cdot \nabla v(x) dx \quad \text{for all } u, v \in W_0^{1,p}(\Omega).$$

In the expression above we have the gradient

$$\nabla u = \left(\frac{\partial u}{\partial x_1}, \dots, \frac{\partial u}{\partial x_N} \right)$$

and the notation $|\nabla u(x)|$ stands for the Euclidean norm of $\nabla u(x) \in \mathbb{R}^N$. In the sequel, the usual norm on any space $L^r(\Omega)$ is denoted $\|\cdot\|_r$. We recall that the first eigenvalue of $-\Delta_p$ on $W_0^{1,p}(\Omega)$ is given by

$$\lambda_1 = \inf_{u \in W_0^{1,p}(\Omega)\setminus\{0\}} \frac{\||\nabla u|\|_p^p}{\|u\|_p^p}.$$

Any other p-Laplacian type operator can be used in our arguments. We refer to [7] for the study of such operators.

The reaction term in (P) is described by a composition involving a Carathéodory function $f : \Omega \times \mathbb{R} \times \mathbb{R}^N \to \mathbb{R}$ (i.e., $f(\cdot, s, \xi)$ is measurable for all $(s, \xi) \in \mathbb{R} \times \mathbb{R}^N$ and $f(x, \cdot, \cdot)$ is continuous for a.e. $x \in \Omega$) and a continuous map (linear or nonlinear) $B : W_0^{1,p}(\Omega) \to W^{1,p}(\Omega)$ that we call intrinsic operator. This special structure of the convection term makes the problem nonstandard and challenging. The study of nonlinear elliptic equations exhibiting intrinsic operator was initiated in [8] and continued in [4, 6, 10, 11].

We understand a solution to problem (P) in the weak sense, that is, any $u \in W_0^{1,p}(\Omega)$ satisfying $f(x, Bu, \nabla(Bu))v \in L^1(\Omega)$ and

$$\int_\Omega |\nabla u|^{p-2} \nabla u \nabla v dx = \int_\Omega f(x, Bu, \nabla(Bu)) v dx$$

for all $v \in W_0^{1,p}(\Omega)$. We seek the solutions to problem (P) in the space $W_0^{1,p}(\Omega)$ due to the homogeneous boundary condition.

The space $W_0^{1,p}(\Omega)$ is endowed with the norm $\|u\|_{W_0^{1,p}} = (\int_\Omega |\nabla u|^p dx)^{\frac{1}{p}}$, while the norm on the space $W^{1,p}(\Omega)$ is

$$\|u\|_{W^{1,p}} = \|u\|_p + \sum_{i=1}^N \left\| \frac{\partial u}{\partial x_i} \right\|_p .$$

The dual space of $W_0^{1,p}(\Omega)$ is $W^{-1,p'}(\Omega)$. In order to simplify the notation, corresponding to any real number r we denote $r' = r/(r-1)$ (the Hölder conjugate of r). In order to avoid repetitive arguments, for the rest of the paper we suppose that $N > p$. The case $N \leq p$ is simpler and can be handled analogously. Therefore, the Sobolev critical exponent is $p^* = \frac{Np}{N-p}$.

Our assumptions on the Carathéodory function $f : \Omega \times \mathbb{R} \times \mathbb{R}^N \to \mathbb{R}$ and the operator $B : W_0^{1,p}(\Omega) \to W^{1,p}(\Omega)$ are as follows:

(H1) There exist a function $\sigma \in L^{r'}(\Omega)$ with $r \in [1, p^*)$, constants $c \geq 0$, $d \geq 0$, $\alpha \in [0, p^* - 1)$, and $\beta \in [0, \frac{p}{(p^*)'})$ such that

$$|f(x, s, \xi)| \leq \sigma(x) + c|s|^\alpha + d|\xi|^\beta$$

for a.e. $x \in \Omega$ and all $(s, \xi) \in \mathbb{R} \times \mathbb{R}^N$.

(H2) The map $B : W_0^{1,p}(\Omega) \to W^{1,p}(\Omega)$ is continuous and there are nonnegative constants K_1 and K_2 such that

$$\|Bu\|_p + \||\nabla(Bu)|\|_p \leq K_1\|u\|_{W_0^{1,p}} + K_2$$

for all $u \in W_0^{1,p}(\Omega)$.

(H3) There exist a function $\zeta \in L^1(\Omega)$ and constants $a \geq 0$, $b \geq 0$ with

$$1 > 2^{p-1}(a+b)K_1^p \tag{1.1}$$

such that

$$f(x,s,\xi)s \leq \zeta(x) + a|s|^p + b|\xi|^p$$

for a.e. $x \in \Omega$ and all $(s,\xi) \in \mathbb{R} \times \mathbb{R}^N$.

An example of a nonlinear term satisfying these assumptions is supplied in Section 5 (see the proof of Theorem 5.1).

We state our main abstract result.

Theorem 1.1. *Assume that conditions* (H1)–(H3) *are verified. Then problem* (P) *admits at least one weak solution. In addition, if* $u \in W_0^{1,p}(\Omega)$ *is a weak solution to problem* (P)*, there is the bound*

$$\|u\|_{W_0^{1,p}} \leq \left(\frac{\|\zeta\|_1 + 2^{p-1}(a+b)K_2^p}{1 - 2^{p-1}(a+b)K_1^p}\right)^{\frac{1}{p}}. \tag{1.2}$$

The proof of Theorem 1.1 is given in Section 4

Estimate (1.2) enables us to get an answer to the fundamental question when a prescribed open ball in the space $W_0^{1,p}(\Omega)$ contains a solution to problem (P). The following sufficient condition is available.

Corollary 1.2. *Assume that conditions* (H1)–(H3) *hold. Given* $R > 0$ *there exists a weak solution* $u \in W_0^{1,p}(\Omega)$ *to problem* (P) *satisfying the a priori estimate* $\|u\|_{W_0^{1,p}} < R$ *provided*

$$R > \left(\frac{\|\zeta\|_1 + 2^{p-1}(a+b)K_2^p}{1 - 2^{p-1}(a+b)K_1^p}\right)^{\frac{1}{p}}.$$

The proof follows readily from Theorem 1.1.

The abstract result formulated in Theorem 1.1 is applied to a quasilinear elliptic problem exhibiting a convolution product. The study of problems of this type involving convolution started with [9]. For related results we refer to [5, 8, 11, 12]. Our application of Theorem 1.1 is presented in Theorem 5.1 which is stated and proven in Section 5.

2. PRELIMINARIES

According to the Sobolev embedding theorem, the space $W_0^{1,p}(\Omega)$ is continuously embedded in $L^\tau(\Omega)$ whenever $\tau \in [1, p^*]$, so there exists a constant $\theta_\tau > 0$ such that

$$\|u\|_\tau \leq \theta_\tau \|u\|_{W_0^{1,p}}, \quad \forall u \in W_0^{1,p}(\Omega). \tag{2.1}$$

Moreover, the Rellich–Kondrachov theorem ensures that the embedding of $W_0^{1,p}(\Omega)$ into $L^\tau(\Omega)$ is compact for $\tau \in [1, p^*)$.

The proof of the following basic result can be found in [2].

Proposition 2.1. *The negative p-Laplacian $-\Delta_p : W_0^{1,p}(\Omega) \to W^{-1,p'}(\Omega)$ is continuous, bounded (in the sense that it maps bounded sets into bounded sets), maximal monotone, strictly monotone (so, pseudomonotone) and satisfies the (S_+)-property, that is, any sequence $\{u_n\} \subset W_0^{1,p}(\Omega)$ for which $u_n \rightharpoonup u$ in $W_0^{1,p}(\Omega)$ and $\limsup_{n\to+\infty} \langle -\Delta_p u_n, u_n - u\rangle \le 0$ fulfills $u_n \to u$ in $W_0^{1,p}(\Omega)$.*

We will also need the following theorem.

Theorem 2.2 ([2, Theorem 2.99])**.** *Let X be a real reflexive Banach space and let $A : X \to X^*$ be a bounded, coercive and pseudomonotone operator. Then for every $b \in X^*$ the equation $Au = b$ has at least one solution $u \in X$.*

A comprehensive presentation of monotonicity methods can be found in [3].

Our application concerns the convolution product, a notion that we now recall within our setting. The convolution of $\rho \in L^1(\mathbb{R}^N)$ and $u \in W_0^{1,p}(\Omega)$, $1 < p < +\infty$, is defined as

$$(\rho * u)(x) = \int_{\mathbb{R}^N} \rho(y)u(x-y)dy \quad \text{for } x \in \Omega,$$

where $u \in W_0^{1,p}(\Omega) \subset W^{1,p}(\mathbb{R}^N)$ is extended by 0 on $\mathbb{R}^N \setminus \Omega$.

By Young's theorem for convolution we have

$$\|\rho * u\|_{L^p(\mathbb{R}^N)} \le \|\rho\|_{L^1(\mathbb{R}^N)}\|u\|_{L^p(\mathbb{R}^N)} = \|\rho\|_{L^1(\mathbb{R}^N)}\|u\|_p \tag{2.2}$$

(see [1, Theorem 4.15]). It is known that

$$\frac{\partial}{\partial x_i}(\rho * u) = \rho * \frac{\partial u}{\partial x_i}, \quad i = 1, \ldots, N \tag{2.3}$$

(see [1, Lemma 9.1]). From (2.2) and (2.3) we obtain

$$\|\rho * u\|_p + \||\nabla(\rho * u)|\|_p \le \|\rho\|_{L^1(\mathbb{R}^N)} \left(\lambda_1^{-\frac{1}{p}} + N\right) \|u\|_{W_0^{1,p}}. \tag{2.4}$$

3. THE OPERATOR EQUATION

Let us introduce the operator $A : W_0^{1,p}(\Omega) \to W^{-1,p'}(\Omega)$ by

$$\langle A(u), v\rangle = \int_\Omega f(x, Bu, \nabla(Bu))v dx \tag{3.1}$$

for all $u, v \in W_0^{1,p}(\Omega)$.

It is apparent from (3.1) that problem (P) can be equivalently expressed as the operator equation

$$-\Delta_p u - A(u) = 0. \tag{3.2}$$

The equality in (3.2) is required in the dual space $W^{-1,p'}(\Omega)$, and we seek the solution u in $W_0^{1,p}(\Omega)$.

The next propositions point out useful properties of the operator A.

Proposition 3.1. *Assume that conditions* (H1)–(H2) *hold. Then for the operator A defined in* (3.1) *there is the following bound*

$$\|A(u)\|_{W^{-1,p'}} \leq \|\sigma\|_{r'}\theta_r + c\theta_{\frac{p^*}{p^*-\alpha}}\|Bu\|_{p^*}^{\alpha} + d\theta_{\frac{p}{p-\beta}}\||\nabla(Bu)|\|_p^{\beta} \tag{3.3}$$

for all $u \in W_0^{1,p}(\Omega)$. In particular, the operator A from $W_0^{1,p}(\Omega)$ to $W^{-1,p'}(\Omega)$ is well defined and bounded.

Proof. By hypothesis (H1) and using (2.1) in conjunction with Hölder's inequality we find that

$$\begin{aligned}\left|\int_\Omega f(x, Bu, \nabla(Bu))v\,dx\right| &\leq \int_\Omega (|\sigma(x)| + c|Bu|^\alpha + d|\nabla(Bu)|^\beta)|v|dx \\ &\leq \|\sigma\|_{r'}\|v\|_r + c\|Bu\|_{p^*}^{\alpha}\|v\|_{\frac{p^*}{p^*-\alpha}} + d\||\nabla(Bu)|\|_p^{\beta}\|v\|_{\frac{p}{p-\beta}} \\ &\leq (\|\sigma\|_{r'}\theta_r + c\theta_{\frac{p^*}{p^*-\alpha}}\|Bu\|_{p^*}^{\alpha} + d\theta_{\frac{p}{p-\beta}}\||\nabla(Bu)|\|_p^{\beta})\|v\|_{W_0^{1,p}}.\end{aligned}$$

We are thus led to the bound in (3.3).

Hypothesis (H2) ensures that the operator A from $W_0^{1,p}(\Omega)$ to $W^{-1,p'}(\Omega)$ is bounded. As the boundary $\partial\Omega$ is Lipschitz continuous there is the continuous embedding of $W^{1,p}(\Omega)$ into $L^{p*}(\Omega)$. Consequently, estimate (3.3) implies that the operator A from $W_0^{1,p}(\Omega)$ to $W^{-1,p'}(\Omega)$ is well defined and bounded, which completes the proof. □

Proposition 3.2. *Assume conditions* (H1)–(H2)*. If $u_n \rightharpoonup u$ in $W_0^{1,p}(\Omega)$, then there holds*

$$\lim_{n\to\infty} \langle A(u_n), u_n - u\rangle = 0. \tag{3.4}$$

Moreover, the operator $-\Delta_p - A : W_0^{1,p}(\Omega) \to W^{-1,p'}(\Omega)$ which drives the equation in (3.2) *satisfies the (S_+)-property.*

Proof. By hypothesis (H1) we see that

$$\left|\int_\Omega f(x, Bu, \nabla(Bu))(u_n - u)dx\right| \leq \int_\Omega (|\sigma| + c|Bu_n|^\alpha + d|\nabla(Bu_n)|^\beta)|u_n - u|dx.$$

Since B is a bounded operator, the sequence $\{Bu_n\}$ is bounded in $W^{1,p}(\Omega)$ and thus in $L^{p^*}(\Omega)$. Then through the Rellich–Kondrachov embedding theorem (note from (H1) that $r < p^*$, $\frac{p^*}{p^*-\alpha} < p^*$, $\frac{p}{p-\beta} < p^*$), we get

$$\int_\Omega |\sigma(x)||u_n - u|dx \leq \|\sigma\|_{r'}\|u_n - u\|_r \to 0 \text{ as } n \to +\infty,$$

$$\int_\Omega |Bu_n|^\alpha |u_n - u|dx \leq \|Bu_n\|_{p^*}^\alpha \|u_n - u\|_{\frac{p^*}{p^*-\alpha}} \to 0 \text{ as } n \to +\infty,$$

and

$$\int_\Omega |\nabla Bu_n|^\beta |u_n - u|dx \leq \|Bu_n\|_{W^{1,p}}^\beta \|u_n - u\|_{\frac{p}{p-\beta}} \to 0 \text{ as } n \to +\infty.$$

In view of the definition of the operator A in (3.1), we arrive at (3.4).

The second part of Proposition 3.2 follows from (3.4) and Proposition 2.1. This completes the proof. □

4. EXISTENCE AND BOUND OF SOLUTIONS

As seen from equation (3.2), in order to resolve problem (P) we have to study the operator $-\Delta_p - A : W_0^{1,p}(\Omega) \to W^{-1,p'}(\Omega)$, where A is given by (3.1). The next lemmas establish essential properties of this operator.

Lemma 4.1. *Assume that conditions* (H1)–(H2) *are satisfied. Then the operator* $-\Delta_p - A$ *is bounded and pseudomonotone.*

Proof. First we prove that the operator $-\Delta_p - A$ is bounded. Indeed, the definition of the negative p-Laplacian $-\Delta_p$, shows that the operator $-\Delta_p$ is bounded. Proposition 3.1 entails that the operator A is bounded. Consequently, $-\Delta_p - A$ is bounded.

We now pass to prove that the operator $-\Delta_p - A$ is pseudomonotone. Specifically, we have to check that if $u_n \rightharpoonup u$ in $W_0^{1,p}(\Omega)$ and

$$\limsup_{n\to\infty} \langle -\Delta_p u_n - A(u_n), u_n - u\rangle \leq 0, \tag{4.1}$$

then

$$\liminf_{n\to\infty} \langle -\Delta_p u_n - A(u_n), u_n - v\rangle \geq \langle -\Delta_p u - A(u), u - v\rangle \tag{4.2}$$

for all $v \in W_0^{1,p}(\Omega)$.

Using (3.4), we note that (4.1) reduces to

$$\limsup_{n\to+\infty} \langle -\Delta_p u_n, u_n - u\rangle \leq 0.$$

Hence, Proposition 2.1 yields the strong convergence $u_n \to u$ in $W_0^{1,p}(\Omega)$. Then the validity of (4.2) readily follows, thus completing the proof. □

Lemma 4.2. *Assume that conditions* (H1)–(H3) *hold. Then the operator* $-\Delta_p - A$, *with* A *defined by* (3.1), *is coercive, that is*

$$\lim_{\|u\|_{W_0^{1,p}}\to\infty} \frac{\langle -\Delta_p u - A(u), u\rangle}{\|u\|_{W_0^{1,p}}} \to +\infty. \tag{4.3}$$

Proof. Hypotheses (H2) and (H3), in conjunction with a well-known convexity inequality, imply

$$\begin{aligned}\int_\Omega f(x, Bu, \nabla(Bu))u dx &\leq \|\zeta\|_1 + a\|Bu\|_p^p + +b\||\nabla(Bu)|\|_p^p \\ &\leq \|\zeta\|_1 + a(K_1\||\nabla u|\|_p + K_2)^p + b(K_1\||\nabla u|\|_p + K_2)^p \\ &\leq \|\zeta\|_1 + 2^{p-1}(a+b)(K_1^p\||\nabla u|\|_p^p + K_2^p).\end{aligned}$$

This results in

$$\langle -\Delta_p u - A(u), u\rangle \geq \left(1 - 2^{p-1}(a+b)K_1^p\right)\||\nabla u_1|\|_p^p - \|\zeta\|_1 - 2^{p-1}(a+b)K_2^p.$$

We conclude that (4.3) holds true because of (1.1). □

We are going to prove Theorem 1.1.

Proof of Theorem 1.1. For the first part of Theorem 1.1, since problem (P) is equivalent to equation (3.2) it is sufficient to check that Theorem 2.2 can be applied to the operator $-\Delta_p - A : W_0^{1,p}(\Omega) \to W^{-1,p'}(\Omega)$, with A defined in (3.1). Lemma 4.1 ensures that the operator $-\Delta_p - A$ is bounded and pseudomonotone, while Lemma 4.2 provides that the operator $-\Delta_p - A$ is coercive. Therefore, Theorem 2.2 can be applied to the operator $-\Delta_p - A$, which guarantees that there exists a solution $u \in W_0^{1,p}(\Omega)$ of equation (3.2). Accordingly, $u \in W_0^{1,p}(\Omega)$ is a weak solution to problem (P).

For the second part of Theorem 1.1, let $u \in W_0^{1,p}(\Omega)$ be a weak solution to problem (P). Testing the equation with the solution u, through hypotheses (H2) and (H3) we find that

$$\||\nabla u|\|_p^P = \int_\Omega f(x, Bu, \nabla(Bu))u dx \leq \|\zeta\|_1 + 2^{p-1}(a+b)K_2^p + 2^{p-1}(a+b)K_1^p\|\nabla u\|_p^p.$$

From here, on the basis of (1.1), it is straightforward to obtain estimate (1.2), which completes the proof of Theorem 1.1. □

5. AN EQUATION WITH CONVOLUTION

In this section we discuss the application of Theorem 1.1 to the following problem with convolution

$$\begin{cases} -\Delta_p u = \dfrac{1}{1+(\rho * u)^2}(1 + |\nabla(\rho * u)|^\gamma) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{C}$$

where $p \in (1, +\infty)$, $\rho \in L^1(\Omega)$ and $\gamma > 0$.

We state our result on problem (C).

Theorem 5.1. *Assume that* $\gamma \in [0, \frac{p}{(p^*)'})$ *and*

$$\|\rho\|_{L^1(\mathbb{R}^N)} < 2^{1-p} \left(\lambda_1^{-\frac{1}{p}} + N \right)^{-1}. \tag{5.1}$$

Then problem (C) *admits at least one weak solution. Moreover, each weak solution* $u \in W_0^{1,p}(\Omega)$ *of problem* (C) *satisfies the estimate*

$$\|u\|_{W_0^{1,p}} \leq \left(\frac{2|\Omega|}{1 - 2^{p-1}\|\rho\|^p_{L^1(\mathbb{R}^N)} \left(\lambda_1^{-\frac{1}{p}} + N \right)^p} \right)^{\frac{1}{p}} \tag{5.2}$$

Proof. Let us show that we can fit in the framework of Theorem 1.1 for problem (P) provided $\|\rho\|_{L^1(\mathbb{R}^N)}$ is sufficiently small as indicated in (5.1). In this respect we introduce $f : \Omega \times \mathbb{R} \times \mathbb{R}^N \to \mathbb{R}$ by

$$f(x, s, \xi) = \frac{1}{1+s^2}(1 + |\xi|^\gamma), \quad \forall (x, s, \xi) \in \Omega \times \mathbb{R} \times \mathbb{R}^N$$

and $B : W_0^{1,p}(\Omega) \to W^{1,p}(\Omega)$ by

$$Bv = \rho * v, \quad \forall v \in W_0^{1,p_i}(\Omega).$$

The assumption on γ ensures that hypothesis (H1) is satisfied. It is seen from (2.4) that hypothesis (H2) is fulfilled by taking $K_1 = \|\rho\|_{L^1(\mathbb{R}^N)}\big(\lambda_1^{-\frac{1}{p}} + N\big)$ and $K_2 = 0$. We also note that

$$f(x, s, \xi)s = \frac{s}{1+s^2}(1 + |\xi|^\gamma) \leq 2 + |\xi|^p,$$

so hypothesis (H3) holds with $\zeta = 2$, $a = 0$, and $b = 1$ because (5.1) was supposed to be true.

As a consequence, Theorem 1.1 can be applied. It turns out that there exists a weak solution $u \in W_0^{1,p_i}(\Omega)$ of problem (C). The bound in (5.2) follows from (1.2). The proof is complete. □

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