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DECOMPOSITIONS OF COMPLETE 3-UNIFORM HYPERGRAPHS INTO CYCLES OF CONSTANT PRIME LENGTH

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Abstract. A complete 3-uniform hypergraph of order n has vertex set V with $|V| = n$ and the set of all 3-subsets of V as its edge set. A t-cycle in this hypergraph is $v_1, e_1, v_2, e_2, \ldots, v_t, e_t, v_1$ where $v_1, v_2, \ldots, v_t$ are distinct vertices and $e_1, e_2, \ldots, e_t$ are distinct edges such that $v_i, v_{i+1} \in e_i$ for $i \in \{1, 2, \ldots, t-1\}$ and $v_t, v_1 \in e_t$. A *decomposition* of a hypergraph is a partition of its edge set into edge-disjoint subsets. In this paper, we give necessary and sufficient conditions for a decomposition of the complete 3-uniform hypergraph of order n into p-cycles, whenever p is prime.

Keywords: uniform hypergraph, cycle decomposition.

Mathematics Subject Classification: 05C65, 05C85.

1. INTRODUCTION

A *hypergraph* $\mathcal{H}$ consists of a finite nonempty set V of *vertices* and a set $\mathcal{E} = \{e_1, e_2, \ldots, e_m\}$ of *e*dges where each $e_i \subseteq V$ with $|e_i| > 0$ for $i \in \{1, 2, \ldots, m\}$. If $|e_i| = h$, then we call e_i an h-*e*dge. If every edge of $\mathcal{H}$ is an h-edge for some h, then we say that $\mathcal{H}$ is h-*u*niform. The *complete* h-*u*niform hypergraph $K_n^{(h)}$ is the hypergraph with vertex set V, where $|V| = n$, in which every h-subset of V determines an h-edge. It then follows that $K_n^{(h)}$ has $\binom{n}{h}$ edges. When $h = 2$, $K_n^{(2)} = K_n$, the complete graph on n vertices.

A *decomposition* of a hypergraph $\mathcal{H}$ is a set $\mathcal{F} = \{\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_k\}$ of *subhypergraphs* of $\mathcal{H}$ such that $\mathcal{E}(\mathcal{F}_1) \cup \mathcal{E}(\mathcal{F}_2) \cup \cdots \cup \mathcal{E}(\mathcal{F}_k) = \mathcal{E}(\mathcal{H})$ and $\mathcal{E}(\mathcal{F}_i) \cap \mathcal{E}(\mathcal{F}_j) = \emptyset$ for all $i, j \in \{1, 2, \ldots, k\}$ with $i \neq j$. We denote this by $\mathcal{H} = \mathcal{F}_1 \oplus \mathcal{F}_2 \oplus \ldots \oplus \mathcal{F}_k$. If $\mathcal{H} = \mathcal{F}_1 \oplus \mathcal{F}_2 \oplus \ldots \oplus \mathcal{F}_k$ is a decomposition such that $\mathcal{F}_1 \cong \mathcal{F}_2 \cong \cdots \cong \mathcal{F}_k \cong \mathcal{G}$, where $\mathcal{G}$ is a fixed hypergraph, then $\mathcal{F}$ is called a $\mathcal{G}$-*d*ecomposition of $\mathcal{H}$.

A *cycle of length* t in a hypergraph $\mathcal{H}$ is a sequence of the form $v_1, e_1, v_2, e_2, \ldots, v_t, e_t, v_1$, where $v_1, v_2, \ldots, v_t$ are distinct vertices and $e_1, e_2, \ldots, e_t$ are distinct edges satisfying $v_i, v_{i+1} \in e_i$ for $i \in \{1, 2, \ldots, t-1\}$ and $v_t, v_1 \in e_t$.

Decompositions of $K_n^{(3)}$ into Hamilton cycles were considered in [2,3] and the proof of their existence was given in [10]. Decompositions of $K_n^{(h)}$ into Hamilton cycles were considered in [6, 8], a complete solution for $h \geq 4$ and $n \geq 30$ was given in [6], and cyclic decompositions were considered in [8]. In [4], necessary and sufficient conditions were given for a $\mathcal{G}$-decomposition of $K_n^{(3)}$, where $\mathcal{G}$ is any 3-uniform hypergraph with at most three edges and at most six vertices. In [5], decompositions of $K_n^{(3)}$ into 4-cycles were considered and their existence were established. In [7], decompositions of $K_n^{(3)}$ into 6-cycles were considered and their existence was given.

In this paper, we are interested in p-cycle decompositions of $K_n^{(3)}$, whenever p is prime. A necessary condition for the existence of a t-cycle decomposition of $K_n^{(3)}$ is: t divides the number of edges in $K_n^{(3)}$, that is, $t|\binom{n}{3}$.

The main result of the paper is as follows:

Theorem 1.1. *If* $t \geq 5$ *is an odd integer,* $t \equiv 1$ *or* $5 \pmod 6$ *and* $n \equiv 0,\ 1$ *or* $2 \pmod t$*, then* $K_n^{(3)}$ *has a* t*-cycle decomposition.*

Corollary 1.2. *If* $p \geq 5$ *is prime, then* $K_n^{(3)}$ *has a* p*-cycle decomposition if and only if* $n \equiv 0,\ 1$ *or* $2 \pmod p$*.*

2. TOOLS

We will assume the vertex set of $K_n^{(3)}$ as $\{v_i : i \in \mathbb{Z}_n\}$, where $\mathbb{Z}_n$ is the set of integers modulo n. For non-negative integers i and j with $i < j$, we denote the set $\{v_i, v_{i+1}, \ldots, v_j\}$ by $[v_i, v_j]$, and the set $\{i, i+1, \ldots, j\}$ by $[i, j]$.

For convenience, we will often write the edge $\{v_a, v_b, v_c\}$ as $v_a - v_b - v_c$ and the t-cycle $v_1, e_1, v_2, e_2, \ldots, v_t, e_t, v_1$ as $(v_1 - y_1 - v_2, v_2 - y_2 - v_3, \ldots, v_t - y_t - v_1)$, where $e_i = v_i - y_i - v_{i+1}$ for $i \in \{1, 2, \ldots, t-1\}$ and $e_t = v_t - y_t - v_1$.

2.1. THE HYPERGRAPH $K_{m,n}^{(3)}$

Define the 3-uniform hypergraph $K_{m,n}^{(3)}$ of order $m+n$ as follows. Let

$$V(K_{m,n}^{(3)}) = \{v_i : i \in \mathbb{Z}_{m+n}\}$$

grouped as $G_0 = [v_0, v_{m-1}]$ and $G_1 = [v_m, v_{m+n-1}]$. Let $\mathcal{E}(K_{m,n}^{(3)})$ be the set of all 3-edges $v_a - v_b - v_c$ such that v_a, v_b and v_c are not all from the same group, that is, at least one of $\{v_a, v_b, v_c\}$ is an element of G_0 and at least one of $\{v_a, v_b, v_c\}$ is an element of G_1. Note that $\mathcal{E}(K_{m,n}^{(3)})| = \frac{mn(m+n-2)}{2}$. A necessary condition for the existence of a t-cycle decomposition of $K_{m,n}^{(3)}$ is that $2t|mn(m+n-2)$.

Lemma 2.1. *If $t \geq 5$ is an odd integer, then $K_{1,t}^{(3)}$ decomposes into t-cycles.*

Proof. The complete graph K_t with vertex set $[v_1, v_t]$ is Hamilton cycle decomposable. For each Hamilton cycle $(x_1, x_2, \dots, x_t, x_1)$ in the Hamilton cycle decomposition of K_t,

$$(v_0 - x_1 - x_2, x_2 - v_0 - x_3, x_3 - v_0 - x_4, \dots, x_{t-1} - v_0 - x_t, x_t - x_1 - v_0)$$

is a t-cycle in $K_{1,t}^{(3)}$. A collection of all these t-cycles yields a decomposition of $K_{1,t}^{(3)}$ into t-cycles. □

Lemma 2.2. *If $t \geq 5$ is an odd integer, then $K_{2,t}^{(3)}$ decomposes into t-cycles.*

Proof. The complete graph K_t with vertex set $[v_2, v_{t+1}]$ is Hamilton cycle decomposable. For convenience relabel the vertex v_2 by u_∞ and the vertices in $[v_3, v_{t+1}]$ by $[u_1, u_{t-1}]$, where the suffixes under u are reduced modulo $t-1$ with residues $1, 2, \dots, t-1$. Now consider the Hamilton cycle decomposition:

$$\Big\{C_j := u_\infty u_{1+j} u_{2+j} u_{t-1+j} u_{3+j} u_{t-2+j} u_{4+j} \dots u_{\frac{t+5}{2}+j} u_{\frac{t-1}{2}+j} u_{\frac{t+3}{2}+j} u_{\frac{t+1}{2}+j} u_\infty : \\ j \in \Big[0, \frac{t-3}{2}\Big]\Big\}.$$

The following are collections of t-cycles in $K_{2,t}^{(3)}$ obtained from C_j's:

$$\Big\{C_j^0 := (u_\infty - v_0 - u_{1+j}, u_{1+j} - v_0 - u_{2+j}, u_{2+j} - v_0 - u_{t-1+j}, \\ u_{t-1+j} - v_0 - u_{3+j}, u_{3+j} - v_0 - u_{t-2+j}, \\ u_{t-2+j} - v_0 - u_{4+j}, \dots, u_{\frac{t+5}{2}+j} - v_0 - u_{\frac{t-1}{2}+j}, u_{\frac{t-1}{2}+j} - v_0 - u_{\frac{t+3}{2}+j}, \\ u_{\frac{t+3}{2}+j} - v_0 - u_{\frac{t+1}{2}+j}, u_{\frac{t+1}{2}+j} - v_0 - u_\infty) : \quad j \in \Big[0, \frac{t-3}{2}\Big]\Big\},$$

$$\Big\{C_j^1 := (u_\infty - v_1 - u_{1+j}, u_{1+j} - v_1 - u_{2+j}, u_{2+j} - v_1 - u_{t-1+j}, \\ u_{t-1+j} - v_1 - u_{3+j}, u_{3+j} - v_1 - u_{t-2+j}, u_{t-2+j} - v_1 - u_{4+j}, \dots, \\ u_{\frac{t+5}{2}+j} - v_1 - u_{\frac{t-1}{2}+j}, u_{\frac{t-1}{2}+j} - v_1 - u_{\frac{t+3}{2}+j}, \\ u_{\frac{t+3}{2}+j} - v_1 - u_{\frac{t+1}{2}+j}, u_{\frac{t+1}{2}+j} - v_1 - u_\infty) : \quad j \in \Big[0, \frac{t-3}{2}\Big]\Big\}.$$

We obtain $C_j^{0'}$ from C_j^0 by replacing the edge $u_{1+j} - v_0 - u_{2+j}$ by $u_{1+j} - v_0 - v_1$; i.e.,

$$\Big\{C_j^{0'} := (u_\infty - v_0 - u_{1+j}, u_{1+j} - v_1 - v_0, v_0 - u_{2+j} - u_{t-1+j}, \\ u_{t-1+j} - v_0 - u_{3+j}, u_{3+j} - v_0 - u_{t-2+j}, u_{t-2+j} - v_0 - u_{4+j}, \dots, \\ u_{\frac{t+5}{2}+j} - v_0 - u_{\frac{t-1}{2}+j}, u_{\frac{t-1}{2}+j} - v_0 - u_{\frac{t+3}{2}+j}, \\ u_{\frac{t+3}{2}+j} - v_0 - u_{\frac{t+1}{2}+j}, u_{\frac{t+1}{2}+j} - v_0 - u_\infty) : \quad j \in \Big[0, \frac{t-3}{2}\Big]\Big\}.$$

We obtain $C_j^{1'}$ from C_j^1 by replacing the edge $u_{\frac{t+3}{2}+j} - v_1 - u_{\frac{t+1}{2}+j}$ by $v_0 - v_1 - u_{\frac{t+1}{2}+j}$; i.e.,

$$\Big\{C_j^{1'} := (u_\infty - v_1 - u_{1+j}, u_{1+j} - v_1 - u_{2+j}, u_{2+j} - v_1 - u_{t-1+j}, \\ u_{t-1+j} - v_1 - u_{3+j}, u_{3+j} - v_1 - u_{t-2+j}, u_{t-2+j} - v_1 - u_{4+j}, \dots, \\ u_{\frac{t+5}{2}+j} - v_1 - u_{\frac{t-1}{2}+j}, u_{\frac{t-1}{2}+j} - u_{\frac{t+3}{2}+j} - v_1, \\ v_1 - v_0 - u_{\frac{t+1}{2}+j}, u_{\frac{t+1}{2}+j} - v_1 - u_\infty) : \quad j \in \Big[0, \frac{t-3}{2}\Big]\Big\}.$$

Observe that

$$\Big\{C_j^{0'},\ C_j^{1'}\ :\ j \in \Big[0, \frac{t-3}{2}\Big]\Big\}$$

forms a collection of $t-1$ edge-disjoint t-cycles in $K_{2,t}^{(3)}$. The edges of $K_{2,t}^{(3)}$ not in these t-cycles are

$$\Big\{u_{1+j} - v_0 - u_{2+j},\ u_{\frac{t+3}{2}+j} - v_1 - u_{\frac{t+1}{2}+j}\ :\ j \in [0, \frac{t-3}{2}]\Big\} \cup \{v_0 - v_1 - u_\infty\}.$$

These edges form the t-cycle

$$(v_1 - u_\infty - v_0, v_0 - u_1 - u_2, u_2 - v_0 - u_3, u_3 - v_0 - u_4, u_4 - v_0 - u_5, \dots, \\ u_{\frac{t-1}{2}} - v_0 - u_{\frac{t+1}{2}}, u_{\frac{t+1}{2}} - v_1 - u_{\frac{t+3}{2}}, u_{\frac{t+3}{2}} - v_1 - u_{\frac{t+5}{2}}, u_{\frac{t+5}{2}} - v_1 - u_{\frac{t+7}{2}}, \dots, \\ u_{t-2} - v_1 - u_{t-1}, u_{t-1} - u_1 - v_1) \quad \text{in } K_{2,t}^{(3)}.$$

This completes the proof. □

Lemma 2.3. *If $t \geq 5$ is an odd integer, then $K_{t,t}^{(3)}$ decomposes into t-cycles.*

Proof. The complete graph K_t is Hamilton cycle decomposable. Let $\mathcal{F}_0$ and $\mathcal{F}_1$ be decompositions of K_t into t-cycles with vertex sets $[v_0, v_{t-1}]$ and $[v_t, v_{2t-1}]$, respectively. For each t-cycle $(x_1, x_2, \dots, x_t, x_1)$ of $\mathcal{F}_0$, construct t edge-disjoint t-cycles

$$(x_1 - v_i - x_2, x_2 - v_i - x_3, x_3 - v_i - x_4, \dots, x_{t-1} - v_i - x_t, x_t - v_i - x_1),$$

where $v_i \in [v_t, v_{2t-1}]$ and for each t-cycle $(y_1, y_2, \dots, y_t, y_1)$ of $\mathcal{F}_1$, construct t edge-disjoint t-cycles

$$(y_1 - v_j - y_2, y_2 - v_j - y_3, y_3 - v_j - y_4, \dots, y_{t-1} - v_j - y_t, y_t - v_j - y_1),$$

where $v_j \in [v_0, v_{t-1}]$. Collection of these t-cycles yield a decomposition of $K_{t,t}^{(3)}$ into t-cycles. □

2.2. THE HYPERGRAPH $Z_{p,q,r}^{(3)}$

Define the 3-uniform hypergraph $Z_{p,q,r}^{(3)}$ of order $p+q+r$ as follows:

$$V(Z_{p,q,r}^{(3)}) = \{v_i : i \in \mathbb{Z}_{p+q+r}\}$$

grouped as $G_0 = [v_0, v_{p-1}]$, $G_1 = [v_p, v_{p+q-1}]$ and $G_2 = [v_{p+q}, v_{p+q+r-1}]$ and let $\mathcal{E}(Z^{(3)}_{p,q,r})$ be the set of all 3-edges $v_a - v_b - v_c$ such that $a \in [0, p-1]$, $b \in [p, p+q-1]$ and $c \in [p+q, p+q+r-1]$. Note that $|\mathcal{E}(Z^{(3)}_{p,q,r})| = pqr$. A necessary condition for the existence of a t-cycle decomposition of $Z^{(3)}_{p,q,r}$ is that $t|pqr$.

Lemma 2.4. *If $t \geq 5$ is an odd integer, then $Z^{(3)}_{t,t,r}$ decomposes into t-cycles.*

To prove this lemma, we need the following theorem.

Theorem 2.5 ([9]). *If m is odd and k divides m, then the complete bipartite graph $K_{m,m}$ has a decomposition into paths of length k.*

*P*roof of Lemma 2.4. By Theorem 2.5, the complete bipartite graph $K_{t,t}$ with bipartition $([v_0, v_{t-1}], [v_t, v_{2t-1}])$ has a decomposition $\mathcal{F}$ into paths of length t. For each path $(x_1, x_2, \ldots, x_t, x_{t+1})$ of length t in $\mathcal{F}$, construct r edge-disjoint t-cycles

$$(v_i - x_1 - x_2, x_2 - v_i - x_3, x_3 - v_i - x_4, x_4 - v_i - x_5, \ldots, x_{t-1} - v_i - x_t, x_t - x_{t+1} - v_i),$$

where $v_i \in [v_{2t}, v_{2t+r-1}]$. This collection of t-cycles yield a decomposition of $Z^{(3)}_{t,t,r}$ into t-cycles. □

Corollary 2.6. *If $t \geq 5$ is an odd integer, then $Z^{(3)}_{t,t,t}$ decomposes into t-cycles.*

Corollary 2.7. *If $t \geq 5$ is an odd integer, then $Z^{(3)}_{t,t,1}$ decomposes into t-cycles.*

3. PROOF OF THE MAIN RESULT

We need the following definition and theorem. A *Hamilton cycle* of a hypergraph $\mathcal{H}$ on n vertices is a cycle of length n.

Theorem 3.1 ([2,3,10]). *If $n \equiv 1,\ 2,\ 4$ or $5 \pmod 6$, then $K^{(3)}_n$ decomposes into Hamilton cycles.*

Decomposition of $K^{(3)}_{t+1}$ from that of $K^{(3)}_t$

Lemma 3.2. *If $t \geq 5$ is an odd integer and $t \equiv 1$ or $5 \pmod 6$, then $K^{(3)}_{t+1}$ decomposes into t-cycles.*

Proof. By Theorem 3.1 and Lemma 2.1, $K^{(3)}_t$ and $K^{(3)}_{1,t}$ are, respectively, t-cycle decomposable and so is $K^{(3)}_{t+1} = K^{(3)}_t \oplus K^{(3)}_{1,t}$, where $V(K^{(3)}_t) = [v_1, v_t]$ and $V(K^{(3)}_{1,t}) = G_0 \cup G_1$, $G_0 = \{v_0\}$, $G_1 = [v_1, v_t]$. □

Decomposition of $K^{(3)}_{t+2}$ from that of $K^{(3)}_t$

Lemma 3.3. *If $t \geq 5$ is an odd integer and $t \equiv 1$ or $5 \pmod 6$, then $K^{(3)}_{t+2}$ decomposes into t-cycles.*

Proof. By Theorem 3.1 and Lemma 2.2, $K_t^{(3)}$ and $K_{2,t}^{(3)}$ are, respectively, t-cycle decomposable and so is $K_{t+2}^{(3)} = K_t^{(3)} \oplus K_{2,t}^{(3)}$, where $V(K_t^{(3)}) = [v_2, v_{t+1}]$ and $V(K_{2,t}^{(3)}) = G_0 \cup G_1$, $G_0 = [v_0, v_1]$, $G_1 = [v_2, v_{t+1}]$. □

Proof of Theorem 1.1. Case 1. $n \equiv 0 \pmod t$

Then $n = kt$ for some positive integer k. We may think of $K_{kt}^{(3)}$ as an edge-disjoint union of k copies of $K_t^{(3)}$, $\frac{k(k-1)}{2}$ copies of $K_{t,t}^{(3)}$ and $\frac{k(k-1)(k-2)}{6}$ copies of $Z_{t,t,t}^{(3)}$. That is,

$$\begin{aligned} K_{kt}^{(3)} = {} & \underbrace{K_t^{(3)} \oplus K_t^{(3)} \oplus \cdots \oplus K_t^{(3)}}_{k\ times} \oplus \underbrace{K_{t,t}^{(3)} \oplus K_{t,t}^{(3)} \oplus \cdots \oplus K_{t,t}^{(3)}}_{\frac{k(k-1)}{2}\ times} \\ & \oplus \underbrace{Z_{t,t,t}^{(3)} \oplus Z_{t,t,t}^{(3)} \oplus \cdots \oplus Z_{t,t,t}^{(3)}}_{\frac{k(k-1)(k-2)}{6}\ times}, \end{aligned}$$

where $V(K_t^{(3)})$, disjoint sets G_0 and G_1 of $K_{t,t}^{(3)}$, and pairwise disjoint sets G_0, G_1 and G_2 of $Z_{t,t,t}^{(3)}$ are in $\{[v_0, v_{t-1}], [v_t, v_{2t-1}], [v_{2t}, v_{3t-1}], \ldots, [v_{(k-1)t}, v_{kt-1}]\}$. As each of the hypergraphs $K_t^{(3)}$, $K_{t,t}^{(3)}$ and $Z_{t,t,t}^{(3)}$ is decomposable into t-cycles by Theorem 3.1, Lemma 2.3 and Corollary 2.6, respectively, we have the required decomposition.

Case 2. $n \equiv 1 \pmod t$

Then $n = kt + 1$ for some positive integer k. We may think of $K_{kt+1}^{(3)}$ as k copies of $K_{t+1}^{(3)}$, $\frac{k(k-1)}{2}$ copies of $K_{t,t}^{(3)}$, $\frac{k(k-1)(k-2)}{6}$ copies of $Z_{t,t,t}^{(3)}$ and $\frac{k(k-1)}{2}$ copies of $Z_{t,t,1}^{(3)}$. That is,

$$\begin{aligned} K_{kt+1}^{(3)} = {} & \underbrace{K_{t+1}^{(3)} \oplus K_{t+1}^{(3)} \oplus \cdots \oplus K_{t+1}^{(3)}}_{k\ times} \oplus \underbrace{K_{t,t}^{(3)} \oplus K_{t,t}^{(3)} \oplus \cdots \oplus K_{t,t}^{(3)}}_{\frac{k(k-1)}{2}\ times} \\ & \oplus \underbrace{Z_{t,t,t}^{(3)} \oplus Z_{t,t,t}^{(3)} \oplus \cdots \oplus Z_{t,t,t}^{(3)}}_{\frac{k(k-1)(k-2)}{6}\ times} \oplus \underbrace{Z_{t,t,1}^{(3)} \oplus Z_{t,t,1}^{(3)} \oplus \cdots \oplus Z_{t,t,1}^{(3)}}_{\frac{k(k-1)}{2}\ times}, \end{aligned}$$

where

$$\begin{aligned} V(K_{t+1}^{(3)}) \in {} & \{[v_0, v_{t-1}] \cup \{v_{kt}\}, [v_t, v_{2t-1}] \cup \{v_{kt}\}, [v_{2t}, v_{3t-1}] \cup \{v_{kt}\}, \\ & \ldots, [v_{(k-1)t}, v_{kt-1}] \cup \{v_{kt}\}\}; \end{aligned}$$

disjoint sets G_0 and G_1 of $K_{t,t}^{(3)}$, pairwise disjoint sets G_0, G_1 and G_2 of $Z_{t,t,t}^{(3)}$, and disjoint sets G_0 and G_1 of $Z_{t,t,1}^{(3)}$ are in $\{[v_0, v_{t-1}], [v_t, v_{2t-1}], [v_{2t}, v_{3t-1}], \ldots, [v_{(k-1)t}, v_{kt-1}]\}$; and the set G_2 of $Z_{t,t,1}^{(3)}$ is $\{v_{kt}\}$. As each of the hypergraphs $K_{t+1}^{(3)}$, $K_{t,t}^{(3)}$, $Z_{t,t,t}^{(3)}$ and $Z_{t,t,1}^{(3)}$ is decomposable into t-cycles by Lemma 3.2, Lemma 2.3, Corollary 2.6 and Corollary 2.7, respectively, we have the required decomposition.

Case 3. $n \equiv 2 \pmod t$

Then $n = kt + 2$ for some positive integer k. We may think of $K^{(3)}_{kt+2}$ as k copies of $K^{(3)}_{t+2}$, $\frac{k(k-1)}{2}$ copies of $K^{(3)}_{t,t}$, $\frac{k(k-1)(k-2)}{6}$ copies of $Z^{(3)}_{t,t,t}$ and $k(k-1)$ copies of $Z^{(3)}_{t,t,1}$. That is,

$$\begin{aligned} K^{(3)}_{kt+2} = & \underbrace{K^{(3)}_{t+2} \oplus K^{(3)}_{t+2} \oplus \cdots \oplus K^{(3)}_{t+2}}_{k \text{ times}} \oplus \underbrace{K^{(3)}_{t,t} \oplus K^{(3)}_{t,t} \oplus \cdots \oplus K^{(3)}_{t,t}}_{\frac{k(k-1)}{2} \text{ times}} \\ & \oplus \underbrace{Z^{(3)}_{t,t,t} \oplus Z^{(3)}_{t,t,t} \oplus \cdots \oplus Z^{(3)}_{t,t,t}}_{\frac{k(k-1)(k-2)}{6} \text{ times}} \oplus \underbrace{Z^{(3)}_{t,t,1} \oplus Z^{(3)}_{t,t,1} \oplus \cdots \oplus Z^{(3)}_{t,t,1}}_{\frac{k(k-1)}{2} \text{ times}} \\ & \oplus \underbrace{Z^{(3)}_{t,t,1} \oplus Z^{(3)}_{t,t,1} \oplus \cdots \oplus Z^{(3)}_{t,t,1}}_{\frac{k(k-1)}{2} \text{ times}}, \end{aligned}$$

where

$$\begin{aligned} V(K^{(3)}_{t+2}) \in \{ & [v_0, v_{t-1}] \cup \{v_{kt}, v_{kt+1}\}, [v_t, v_{2t-1}] \cup \{v_{kt}, v_{kt+1}\}, [v_{2t}, v_{3t-1}] \cup \{v_{kt}, v_{kt+1}\}, \\ & \ldots, [v_{(k-1)t}, v_{kt-1}] \cup \{v_{kt}, v_{kt+1}\}\}; \end{aligned}$$

disjoint sets G_0 and G_1 of $K^{(3)}_{t,t}$, pairwise disjoint sets G_0, G_1 and G_2 of $Z^{(3)}_{t,t,t}$, and disjoint sets G_0 and G_1 of $Z^{(3)}_{t,t,1}$ are in $\{[v_0, v_{t-1}], [v_t, v_{2t-1}], [v_{2t}, v_{3t-1}], \ldots, [v_{(k-1)t}, v_{kt-1}]\}$; the set G_2 of the first $\frac{k(k-1)}{2}$ copies $Z^{(3)}_{t,t,1}$ is $\{v_{kt}\}$; and the set G_2 of the last $\frac{k(k-1)}{2}$ copies $Z^{(3)}_{t,t,1}$ is $\{v_{kt+1}\}$. As each of the hypergraphs $K^{(3)}_{t+2}$, $K^{(3)}_{t,t}$, $Z^{(3)}_{t,t,t}$ and $Z^{(3)}_{t,t,1}$ is decomposable into t-cycles by Lemma 3.3, Lemma 2.3, Corollary 2.6 and Corollary 2.7, respectively, we have the required decomposition. □

Proof of Corollary 1.1. Follows from: (i) $p \geq 5$ is prime and $p | \binom{n}{3}$ implies $n \equiv 0$, 1 or 2 $\pmod p$, (ii) p is prime implies $p \equiv 1$ or 5 $\pmod 6$, and (iii) Theorem 1.1. □

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