DECOMPOSING
COMPLETE 3-UNIFORM HYPERGRAPH $K^{(3)}_n$
INTO 7-CYCLES

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Abstract. We use the Katona–Kierstead definition of a Hamiltonian cycle in a uniform hypergraph. A decomposition of complete $k$-uniform hypergraph $K^{(k)}_n$ into Hamiltonian cycles was studied by Bailey–Stevens and Meszka–Rosa. For $n \equiv 2, 4, 5 \pmod{6}$, we design an algorithm for decomposing the complete 3-uniform hypergraphs into Hamiltonian cycles by using the method of edge-partition. A decomposition of $K^{(3)}_n$ into 5-cycles has been presented for all admissible $n \leq 17$, and for all $n = 4m + 1$ when $m$ is a positive integer. In general, the existence of a decomposition into 5-cycles remains open. In this paper, we show if $42 \mid (n-1)(n-2)$ and if there exist $\lambda = \frac{1}{42}(n^2-1)$ sequences $(k_{i_0}, k_{i_1}, \ldots, k_{i_n})$ on $D_{all}(n)$, then $K^{(3)}_n$ can be decomposed into 7-cycles. We use the method of edge-partition and cycle sequence. We find a decomposition of $K^{(3)}_{37}$ and $K^{(3)}_{43}$ into 7-cycles.

Keywords: uniform hypergraph, 7-cycle, cycle decomposition.

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1. INTRODUCTION

The notion of hamiltonicity in hypergraphs was introduced by Berge in 1970. A problem on decomposing complete 3-uniform hypergraphs into Hamilton cycles has been completely solved by Verall in 1994 [12]. It is also worth to mention a definition of a cycle by Berge. Recently, in [5] there were given necessary and sufficient conditions for a decomposition of the complete 3-uniform hypergraph of order $n$ into 4-cycles. Meanwhile, many papers studied the two different definitions of a Hamiltonian cycle in [6,13], which are due to Katona and Kierstead, Wang and Lee, respectively. In fact, the two different definitions of a Hamiltonian cycle are the same. A decomposition of complete $k$-uniform hypergraphs into Hamiltonian cycles has been considered in [1,3,4,6,10,13–15]. In the paper [10], Hamiltonian decompositions of $K^{(3)}_n$ for all admissible $n \leq 32$ has been resolved. Recently, by programming, using the method of edge-partition and cycle
sequence, we have obtained some results for all admissible $32 < n \leq 46$ and $n \neq 43$ in [2]. Meszka and Rosa have introduced a necessary condition for the existence of 5-cycles such a decomposition is that $n \equiv 1, 2, 5, 7, 10$ or $11 \pmod{15}$. In [10], the problem of decomposing the complete 3-uniform hypergraph into 5-cycles and $\ell(\geq 5)$-cycles are open. A decomposition of $K_n^{(3)}$ into 5-cycles exists for all admissible $n < 17$, for all $n = 4^m + 1$, $m$ a positive integer. In [7, 8], the author proposed to find a decomposition of $K_n^{(3)}$ into 5-cycles for $n \in \{5, 7, 10, 11, 16, 17, 20, 22, 26\}$ and shown that if $K_n^{(3)}$ can be decomposed into 5-cycles, then $K_n^{(3)}$ can also be decomposed into 5-cycles. In [9], we found a decomposition of $K_n^{(3)}$ into 7-cycles for $n \in \{7, 8, 14, 16, 22, 23\}$ and have shown if $K_n^{(3)}$ can be decomposed into 7-cycles, then $K_n^{(3)}$ can also be decomposed into 7-cycles. However, the problem of decomposing the complete 3-uniform hypergraph into 7-cycles are difficult. In this paper, under some conditions, we show if $42 \mid (n - 1)(n - 2)$ and if there exist $\lambda = \frac{(n - 1)(n - 2)}{42}$ sequences $(k_{i_0}, k_{i_1}, \ldots, k_{i_\lambda})$ on $D_{all}(n)$, then $K_n^{(3)}$ can be decomposed into 7-cycles.

2. PRELIMINARIES

A hypergraph $H = (V, E)$ consists of a finite set $V$ of vertices with a family $E$ of subsets of $V$, called hyperedges (or simply edges). If each (hyper)edge has a size $k$, we say that $H$ is a $k$-uniform hypergraph. In particular, the complete $k$-uniform hypergraph on $n$ vertices has all $k$-subsets of $\mathbb{Z}_n = \{0, 1, \ldots, n - 1\}$ as edges, denote it by $K_n^{(k)}$. The set of (hyper)edges of $K_n^{(3)}$ is denoted by $\varepsilon(K_n^{(3)})$.

**Definition 2.1.** Let $H = (V, E)$ be a $k$-uniform hypergraph. An $\ell$-cycle in $H$ is a cyclic ordering $(v_0, v_1, \ldots, v_{\ell - 1})$ of the elements of $V$ such that each mod $k$ consecutive $k$-tuple of vertices is an edge of $H$, where $3 \leq k < \ell - 1$.

**Definition 2.2.** An $\ell$-cycle decomposition of $H$ is a partition of the set of (hyper)edges of $H$ into mutually-edge-disjoint $\ell$-cycles.

We introduce the method of edge-partition and the cycle-model from [3].

**Definition 2.3** ([1]). Let $T = \{a, b, c\}$ be a triple of distinct elements of $\mathbb{Z}_n$. Then its difference pattern $\pi(T)$ is the equivalence class of ordered triples containing cyclic rotations of $(b - a, c - b, a - c)$ and $(c - a, b - c, a - b)$ (where the differences are taken modulo $n$).

Clearly, the three differences sum to zero. Therefore, if we know that the first two differences are $x$ and $y$, then the third is $n - x - y$. Omitting the third number, we obtain a difference pair. Using edge-partition of $K_n^{(3)}$ as in paper [3], all difference pairs of the hypergraph $K_n^{(3)}$ may be obtained. Let $\mathbb{Z}$ be the set of integers, $n$ be a fixed positive integer, and $\mathbb{Z}_n = \{0, 1, \ldots, n - 1\}$. Let

$$D_{all}(n) = \{(k_1, k_2) \mid 1 \leq k_1, k_2 \leq n - 1, \text{ and } k_1 + k_2 \neq n\},$$

$$D(n) = D_e \cup D_l \cup D_m,$$
where
\[ D_c = \{ (k_1, k_2) \in D_{all}(n) \mid k_1 = k_2 = k, 1 \leq k < \frac{n}{2} \}, \]
\[ D_l = \{ (k_1, k_2) \in D_{all}(n) \mid 1 \leq k_1 < k_2 < \frac{n-k_1}{2} \}, \]
\[ D_m = \{ (k_2, k_1) \in D_{all}(n) \mid (k_1, k_2) \in D_l \}. \]

Given a difference pair \((k_1, k_2) \in D_{all}(n)\) and an integer \(m \in \mathbb{Z}_n\), define a subhypergraph of \(K_n^{(3)}\) generated by \((k_1, k_2)\) as follows:
\[ E(m; k_1, k_2) = \{ m, m + k_1, m + k_1 + k_2 \} \pmod{n}, \]
We introduce the notation
\[ H(k_1, k_2) = \{ E(m; k_1, k_2) \mid m \in \mathbb{Z}_n \}, \]
where the addition is performed modulo \(n\).

Now we repeat some of the results from [3] to make the paper self-contained.

**Lemma 2.4 ([3]).** Let \((k_1, k_2)\) and \((k'_1, k'_2)\) be arbitrary two distinct difference pairs in \(D_{all}(n)\) we have either
\[ H(k_1, k_2) \cap H(k'_1, k'_2) = \emptyset \]
or
\[ H(k_1, k_2) = H(k'_1, k'_2) \]
and a necessary and sufficient condition for the second equation is
\[ (k_1, k_2) \equiv \begin{cases} (k'_1, k'_2) & \text{or} \\ (k'_1 + k_2, -k'_2) & \text{or} \\ (-k'_1, k'_1 + k'_2) & \text{or} \\ (k'_2, -k'_1 - k'_2) & \text{or} \\ (-k'_1 - k_2, k'_1) & \text{or} \\ (-k'_2, -k'_1) & \text{mod } n. \end{cases} \]

**Definition 2.5 ([3]).** Let \((k_1, k_2)\) and \((k'_1, k'_2)\) be arbitrary two distinct difference pairs in \(D_{all}(n)\). We say \((k_1, k_2)\) and \((k'_1, k'_2)\) are equivalent if \(H(k_1, k_2) = H(k'_1, k'_2)\). This is denoted by \((k'_1, k'_2) \sim (k_1, k_2)\).

**Lemma 2.6** (Edge-partition of \(K_n^{(3)}\), [3]). For any \(K_n^{(3)}\),
\[ e(K_n^{(3)}) = \bigcup_{(k_1, k_2) \in D(n)} H(k_1, k_2), \]
where \((k_1, k_2) \in D(n)\). If \(k_1 \neq k_2\), for convenience, we use \((k_1, k_2)\) to denote \((k_1, k_2)\) and \((k_2, k_1)\).
Definition 2.7. Let \( n \) be a positive integer. For any \( 0 \leq i, j \leq \ell - 1 \), \((k_i, k_{i+1})\) is the edge of the \( H(k_0, k_1, \ldots, k_{\ell-1}) \) as follows:

\[
\sum_{j=0}^{i-1} k_j m + \sum_{j=0}^{i} k_j m + \sum_{j=0}^{i+1} k_j \quad (\text{mod } n).
\]

Sequence (2.1) satisfies the following two conditions:

(a) \( r_0 = 0, \sum_{i=0}^{j} k_i \equiv r_j \pmod n \), \( r_\ell = 0 \).

(b) For any \( i, j \ (i \neq j) \), \( r_i \neq r_j \).

Then \((r_0, r_1, \ldots, r_{\ell-1})\) is an \( \ell \)-cycle, denoted by \( C_\ell = (r_0, r_1, \ldots, r_{\ell-1}) \), called base cycle. According to the definition of difference pattern \( \pi(T) \), we obtain the set of \( \ell \)-cycles \( \{C_\ell + i \mid i \in \mathbb{Z}_n\} \), where \( C_\ell + i = (r_0 + i, r_1 + i, \ldots, r_{\ell-1} + i) \pmod n \). In particular, if \( \ell = n \), then \((r_0, r_1, \ldots, r_{n-1})\) is a base Hamiltonian cycle, denoted by \( C_n = (r_0, r_1, \ldots, r_{n-1}) \).

Definition 2.8. Let \( n \) be a positive integer, for any \( 0 \leq i, j \leq \ell - 1 \), \((k_i, k_{i+1})\) is the edge of the \( D_{\text{all}}(n) \), and \((k_i, k_{i+1}) \neq (k_j, k_{j+1}) \ (i \neq j) \). Given a sequence \((k_0, k_1, \ldots, k_{\ell-1})\) in \( D_{\text{all}}(n) \) and an integer \( m \in \mathbb{Z}_n \), define a subhypergraph of \( K_n^{(3)} \) generated by \((k_0, k_1, \ldots, k_{\ell-1})\) as follows:

\[
(m; k_0, k_1, \ldots, k_{\ell-1}) = \left\{ m, m + k_0, m + k_0 + k_1, \ldots, \sum_{j=0}^{\ell-1} k_j \right\} \pmod n,
\]

\[
E(m; k_0, k_1, \ldots, k_{\ell-1}) = \left\{ m + \sum_{j=0}^{i-1} k_j, m + \sum_{j=0}^{i} k_j, m + \sum_{j=0}^{i+1} k_j \right\} \pmod n,
\]

where \( k_\ell = k_0 \).

We introduce the notation

\[
H(k_0, k_1, \ldots, k_{\ell-1}) = \{E(m; k_0, k_1, \ldots, k_{\ell-1}) \mid m \in \mathbb{Z}_n\},
\]

where the addition is performed modulo \( n \).

Lemma 2.9. Let \( n \) be a positive integer, for any \( 0 \leq i, j \leq \ell - 1 \), \((k_i, k_{i+1})\) is the edge of the \( D_{\text{all}}(n) \), and \((k_i, k_{i+1}) \neq (k_j, k_{j+1}) \ (i \neq j) \). If \((k_0, k_1, \ldots, k_{\ell-1})\) is a sequence on \( D_{\text{all}}(n) \), then

\[
H(k_0, k_1, \ldots, k_{\ell-1}) = \bigcup_{i=0}^{\ell-1} H(k_i, k_{i+1}),
\]

where \( k_\ell = k_0 \).

Proof. For any \( 0 \leq i \leq \ell - 1 \) and an integer \( m \in \mathbb{Z}_n \), there is the edge of the \( H(k_0, k_1, \ldots, k_{\ell-1}) \) as follows:

\[
\left\{ m + \sum_{j=0}^{i-1} k_j, m + \sum_{j=0}^{i} k_j, m + \sum_{j=0}^{i+1} k_j \right\} \pmod n.
\]
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Obviously, the edge $\{m + \sum_{j=0}^{i-1} k_{j}, m + \sum_{j=0}^{i} k_{j}, m + \sum_{j=0}^{i+1} k_{j}\}$ (mod $n$) induces the difference pair $(k_{i}, k_{i+1})$, that is

$$H(k_{i}, k_{i+1}) = \left\{m + \sum_{j=0}^{i-1} k_{j}, m + \sum_{j=0}^{i} k_{j}, m + \sum_{j=0}^{i+1} k_{j}\right\} \pmod{n}.$$  

Hence, we have

$$H(k_{0}, k_{1}, \ldots, k_{\ell-1}) = \bigcup_{i=0}^{\ell-1} H(k_{i}, k_{i+1}).$$ \(\square\)

3. DECOMPOSING $K_n^{(3)}$ INTO 7-CYCLES

**Theorem 3.1.** A necessary condition for the decomposition of complete 3-uniform hypergraph $K_n^{(3)}$ into 7-cycles is $42 \mid n(n-1)(n-2)$.

**Theorem 3.2.** If there exist $\lambda = \frac{(n-1)(n-2)}{42}$ sequences $(k_{i_0}, k_{i_1}, \ldots, k_{i_6})$ on $D_{all}(n)$ by Definition 2.5, then the sequences $(k_{i_0}, k_{i_1}, \ldots, k_{i_6})$ satisfy the following condition: if $(k_{i_0}, k_{i_0+1}) \neq (k_{j_0}, k_{j_0+1})$ $(i \neq j, \alpha \neq \beta, \alpha, \beta \in [0, 5])$, then $K_n^{(3)}$ can be decomposed into 7-cycles.

**Proof.** Let there exist $\lambda = \frac{(n-1)(n-2)}{42}$ sequences $(k_{i_0}, k_{i_1}, \ldots, k_{i_6})$ on $D_{all}(n)$ by Definition 2.5. The sequence $(k_{i_0}, k_{i_1}, \ldots, k_{i_6})$ induces the cycle sequence

$$(r_{i_0}, r_{i_1}, \ldots, r_{i_6}), \quad 1 \leq i \leq \lambda. \quad (3.1)$$

Sequence (3.1) satisfies the following two conditions:

(a) $r_{i_0} = 0, \sum_{\alpha=0}^{i} k_{i_\alpha} \equiv r_{i_1} \pmod{n}, \quad r_{i_7} = 0$.

(b) For any $\alpha, \beta \in [0, 6]$ $(\alpha \neq \beta), r_{i_\alpha} \neq r_{i_\beta}$.

Obviously, we obtain the set of base cycle $\lambda$ 7 cycles. We can decompose the edges of $K_n^{(3)}$ into $n\lambda$ 7-cycles produced by $\lambda$-base 7-cycles. By the structure of base cycle, we obtain the set of 7-cycles $\{C_{7\alpha} + i \mid i \in \mathbb{Z}_n, \alpha \in [1, \lambda]\}$. By the method of edge-partition, we obtain a decomposition of $K_n^{(3)}$ into $n\lambda$ 7-cycles, that is,

$$\varepsilon(K_n^{(3)}) = \bigcup_{(k_1, k_2) \in (D(n))} H(k_1, k_2) = \bigcup_{(k_{i_0}, k_{i_1}, \ldots, k_{i_6}) \in D_{all}(n)} H(k_{i_0}, k_{i_1}, \ldots, k_{i_6})$$

$$= \bigcup_{i=1}^{\lambda} \{C_{7i} + j, \quad j \in \mathbb{Z}_n\}.$$  

Hence, we obtain a decomposition of $K_n^{(3)}$ into $n\lambda$ 7-cycles. \(\square\)
Example 3.3. $K_{37}^{(3)}$ can be decomposed into 7-cycles.

Proof. We can decompose the edges of $K_{37}^{(3)}$ into 1110 7-cycles produced by 30 base 7-cycles as follows. According to our method, we have

$$|\varepsilon(K_{37}^{(3)})| = (\frac{37}{3}) = 7770$$

edges and 7 | 7770. We have

$$D(37) = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6), (7,7), (8,8), (9,9), (10,10), (11,11), (12,12), (13,13), (14,14), (15,15), (16,16), (17,17), (18,18), (1,2), (1,3), (1,4), (1,5), (1,6), (1,7), (1,8), (1,9), (1,10), (1,11), (1,12), (1,13), (1,14), (1,15), (1,16), (1,17), (2,3), (2,4), (2,5), (2,6), (2,7), (2,8), (2,9), (2,10), (2,11), (2,12), (2,13), (2,14), (2,15), (2,16), (2,17), (3,4), (3,5), (3,6), (3,7), (3,8), (3,9), (3,10), (3,11), (3,12), (3,13), (3,14), (3,15), (3,16), (4,5), (4,6), (4,7), (4,8), (4,9), (4,10), (4,11), (4,12), (4,13), (4,14), (4,15), (4,16), (5,6), (5,7), (5,8), (5,9), (5,10), (5,11), (5,12), (5,13), (5,14), (5,15), (6,7), (6,8), (6,9), (6,10), (6,11), (6,12), (6,13), (6,14), (6,15), (7,8), (7,9), (7,10), (7,11), (7,12), (7,13), (7,14), (8,9), (8,9), (8,10), (8,11), (8,12), (8,13), (8,14), (9,10), (9,11), (9,12), (9,13), (10,11), (10,12), (10,13), (11,12)\}.$$

Now, we find the decomposition of $K_{37}^{(3)}$. On $D(37)$, according to Definition 2.5, we obtain $\lambda = 30$ sequences as follows:

(1) (1,1,2,1,3,1,28),  
(2) (1,4,35,5,31,5,30),  
(3) (1,7,33,6,8,2,25),  
(4) (1,9,30,4,1,10,27),  
(5) (1,12,27,2,9,1,22),  
(6) (1,13,26,3,2,12,17),  
(7) (1,15,24,3,3,12,16),  
(8) (1,16,23,3,5,8,18),  
(9) (1,21,18,2,15,4,13),  
(10) (1,24,15,2,16,5,11),  
(11) (1,26,14,4,29,31,7),  
(12) (2,18,22,16,6,16,14),  
(13) (2,23,17,3,6,14,9),  
(14) (2,28,12,28,30,5,6),  
(15) (3,7,3,26,4,10,21),  
(16) (3,14,28,11,9,10,16),  
(17) (3,16,25,31,10,11,15),  
(18) (3,21,20,18,11,13,12),  
(19) (3,23,18,32,8,17,20),  
(20) (4,5,4,25,4,21,11),  
(21) (4,16,6,22,4,23,9),  
(22) (4,9,7,18,5,24,7),  
(23) (4,14,30,27,24,32,5),  
(24) (5,5,7,22,5,15,15),  
(25) (5,18,26,28,17,7,10),  
(26) (5,20,28,27,18,6,7),  
(27) (6,11,12,9,15,8,13),  
(28) (6,19,26,17,13,14,16),  
(29) (6,21,8,9,9,13,8),  
(30) (7,19,25,27,25,23,14),
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Let $D$ be a collection of 30 sequences above. Thus, they correspond to 30-base 7-cycles:

$C_{7(1)} = (0, 1, 2, 4, 5, 8, 9), \quad C_{7(2)} = (0, 1, 5, 3, 8, 2, 7),$

$C_{7(3)} = (0, 1, 8, 4, 2, 10, 12), \quad C_{7(4)} = (0, 1, 10, 3, 7, 8, 18),$

$C_{7(5)} = (0, 1, 13, 3, 5, 14, 15), \quad C_{7(6)} = (0, 1, 14, 3, 6, 8, 20),$

$C_{7(7)} = (0, 1, 16, 3, 6, 9, 21), \quad C_{7(8)} = (0, 1, 17, 3, 6, 11, 19),$

$C_{7(9)} = (0, 1, 22, 3, 5, 20, 24), \quad C_{7(10)} = (0, 1, 25, 3, 5, 21, 26),$

$C_{7(11)} = (0, 1, 27, 3, 7, 36, 30), \quad C_{7(12)} = (0, 2, 20, 5, 1, 7, 23),$

$C_{7(13)} = (0, 2, 25, 5, 814, 28), \quad C_{7(14)} = (0, 2, 30, 5, 33, 26, 31),$

$C_{7(15)} = (0, 3, 10, 13, 2, 6, 16), \quad C_{7(16)} = (0, 3, 17, 8, 2, 11, 21),$

$C_{7(17)} = (0, 3, 19, 7, 1, 11, 22), \quad C_{7(18)} = (0, 3, 24, 7, 1, 12, 25),$

$C_{7(19)} = (0, 3, 26, 7, 2, 10, 27), \quad C_{7(20)} = (0, 4, 9, 13, 1, 5, 26),$

$C_{7(21)} = (0, 4, 10, 16, 1, 5, 28), \quad C_{7(22)} = (0, 4, 13, 20, 1, 6, 30),$

$C_{7(23)} = (0, 4, 18, 11, 1, 25, 20), \quad C_{7(24)} = (0, 5, 10, 17, 2, 7, 22),$

$C_{7(25)} = (0, 5, 23, 12, 3, 20, 27), \quad C_{7(26)} = (0, 5, 25, 16, 6, 24, 30),$

$C_{7(27)} = (0, 6, 17, 29, 1, 16, 24), \quad C_{7(28)} = (0, 6, 25, 14, 31, 7, 21),$

$C_{7(29)} = (0, 6, 27, 35, 7, 16, 29), \quad C_{7(30)} = (0, 7, 26, 14, 4, 29, 15),$

By the method of edge-partition, we obtain the decomposition of $K^{(3)}_{37}$ into 1110 7-cycles, that is

$$
\varepsilon(K^{(3)}_{37}) = \bigcup_{(k_1, k_2) \in D^{(37)}} H(k_1, k_2)
= \bigcup_{(k_0, k_1, \ldots, k_5) \in D_{37}^{(37)}} H(k_0, k_1, \ldots, k_5)
= \bigcup_{i=1}^{30} \{C_{7(i)} + j, \ j \in \mathbb{Z}_{37}\}.
$$

Hence, we obtain the decomposition of $K^{(3)}_{37}$ into 1110 7-cycles. \hfill \Box

**Example 3.4.** $K^{(3)}_{43}$ can be decomposed into 7-cycles.

**Proof.** We can decompose the edges of $K^{(3)}_{43}$ into 1763 7-cycles produced by 41 base 7-cycles as follows. According to our method, we have

$$
|\varepsilon(K^{(3)}_{43})| = \binom{43}{3} = 12341
$$

edges and 7 | 12341.
Now, we need to find the decomposition of $K^{(3)}_{43}$. On $D(43)$, according to Definition 2.5, we obtain 41 sequences as follows:

$D(43) = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6), (7,7), (8,8), (9,9), (10,10), (11,11), (12,12), \ldots\}$

(1) (1, 1, 2, 1, 3, 1, 34),
(2) (1, 4, 41, 5, 37, 5, 36),
(3) (1, 7, 39, 41, 8, 2, 31),
(4) (1, 9, 36, 4, 1, 10, 25),
(5) (1, 12, 33, 2, 9, 1, 28),
(6) (1, 13, 32, 3, 2, 12, 23),
(7) (1, 15, 30, 3, 12, 22),
(8) (1, 16, 29, 3, 5, 8, 24),
(9) (1, 21, 24, 2, 15, 3, 20),
(10) (1, 23, 22, 3, 6, 13, 18),
(11) (1, 25, 20, 4, 2, 18, 16),
(12) (1, 27, 18, 3, 7, 17, 13),
(13) (1, 30, 15, 2, 16, 11, 11),
(14) (1, 32, 13, 4, 35, 37, 7),
(15) (2, 20, 26, 39, 9, 14, 19),
(16) (2, 24, 22, 39, 10, 18, 14),
(17) (22, 29, 17, 3, 9, 20, 6),
(18) (2, 32, 14, 4, 32, 38, 7),
(19) (3, 13, 24, 37, 9, 7, 26),
(20) (3, 15, 32, 37, 10, 19, 13),
(21) (3, 16, 31, 7, 11, 10, 21),
(22) (3, 24, 23, 37, 12, 20, 10),
(23) (3, 28, 19, 25, 13, 21, 8),
(24) (3, 31, 16, 38, 8, 26, 7),
(25) (4, 7, 5, 28, 5, 16, 23),
(26) (4, 8, 4, 28, 7, 6, 29),
(27) (4, 13, 38, 8, 5, 18, 17),
(28) (4, 14, 34, 35, 15, 6, 21),
(29) (4, 15, 37, 7, 17, 9, 16),
(30) (4, 29, 19, 36, 20, 12, 9),
(31) (5, 5, 6, 10, 23, 15, 22),
(32) (5, 7, 7, 25, 5, 19, 18),
(33) (5, 12, 8, 19, 6, 23, 13),
(34) (5, 15, 7, 18, 8, 16, 17),
(35) (5, 17, 7, 16, 35, 33, 16),
(36) (6, 16, 15, 8, 22, 8, 11),
(37) (7, 15, 20, 13, 16, 34, 24),
(38) (7, 20, 34, 17, 27, 29, 22),
(39) (7, 22, 38, 29, 28, 30, 25),
(40) (9, 9, 11, 15, 18, 11, 13),
(41) (9, 10, 11, 17, 12, 12, 15).
Let $D$ be a collection of the 41 sequences above. Thus they correspond to 41 base 7-cycles:

\[ C_{7(1)} = (0, 1, 2, 4, 5, 8, 9), \quad C_{7(2)} = (0, 1, 5, 3, 8, 2, 7), \]
\[ C_{7(3)} = (0, 1, 8, 4, 2, 10, 12), \quad C_{7(4)} = (0, 1, 10, 3, 7, 8, 18), \]
\[ C_{7(5)} = (0, 1, 13, 3, 5, 14, 15), \quad C_{7(6)} = (0, 1, 14, 3, 6, 8, 20), \]
\[ C_{7(7)} = (0, 1, 16, 3, 6, 9, 21), \quad C_{7(8)} = (0, 1, 17, 3, 6, 11, 19), \]
\[ C_{7(9)} = (0, 1, 22, 3, 5, 20, 23), \quad C_{7(10)} = (0, 1, 24, 3, 6, 12, 25), \]
\[ C_{7(11)} = (0, 1, 26, 3, 7, 9, 2), \quad C_{7(12)} = (0, 1, 28, 3, 6, 13, 30), \]
\[ C_{7(13)} = (0, 1, 31, 3, 5, 21, 32), \quad C_{7(14)} = (0, 1, 33, 3, 7, 42, 36), \]
\[ C_{7(15)} = (0, 2, 22, 5, 1, 10, 24), \quad C_{7(16)} = (0, 2, 26, 5, 1, 11, 29), \]
\[ C_{7(17)} = (0, 2, 31, 5, 8, 17, 37), \quad C_{7(18)} = (0, 2, 34, 5, 9, 41, 36), \]
\[ C_{7(19)} = (0, 3, 16, 7, 1, 10, 17), \quad C_{7(20)} = (0, 3, 18, 7, 1, 11, 30), \]
\[ C_{7(21)} = (0, 3, 19, 7, 1, 12, 22), \quad C_{7(22)} = (0, 3, 27, 7, 1, 13, 33), \]
\[ C_{7(23)} = (0, 3, 31, 7, 1, 14, 35), \quad C_{7(24)} = (0, 3, 34, 7, 2, 10, 36), \]
\[ C_{7(25)} = (0, 4, 11, 16, 1, 6, 20), \quad C_{7(26)} = (0, 4, 12, 16, 1, 8, 14), \]
\[ C_{7(27)} = (0, 4, 17, 12, 3, 8, 26), \quad C_{7(28)} = (0, 4, 18, 9, 1, 16, 22), \]
\[ C_{7(29)} = (0, 4, 19, 13, 1, 18, 27), \quad C_{7(30)} = (0, 4, 33, 9, 2, 22, 34), \]
\[ C_{7(31)} = (0, 5, 10, 16, 26, 6, 21), \quad C_{7(32)} = (0, 5, 12, 19, 1, 6, 25), \]
\[ C_{7(33)} = (0, 5, 17, 25, 1, 7, 30), \quad C_{7(34)} = (0, 5, 20, 27, 2, 10, 26), \]
\[ C_{7(35)} = (0, 5, 22, 29, 2, 37, 27), \quad C_{7(36)} = (0, 6, 22, 37, 2, 24, 32), \]
\[ C_{7(37)} = (0, 7, 22, 42, 12, 28, 19), \quad C_{7(38)} = (0, 7, 27, 18, 8, 35, 21), \]
\[ C_{7(39)} = (0, 7, 29, 17, 3, 31, 18), \quad C_{7(40)} = (0, 9, 18, 29, 1, 19, 30), \]
\[ C_{7(41)} = (0, 9, 19, 30, 4, 16, 28). \]

By the method of edge-partition, we obtain the decomposition of $K_{43}^{(3)}$ into 1763 7-cycles, that is,

\[
\varepsilon(K_{43}^{(3)}) = \bigcup_{(k_1, k_2) \in D(43)} H(k_1, k_2) = \bigcup_{(k_0, k_1, \ldots, k_6) \in D_{43}(43)} H(k_0, k_1, \ldots, k_6)
\]

\[
= \bigcup_{i=1}^{41} \{ C_{7(i)} + j, \ j \in \mathbb{Z}_{43} \}.
\]

Hence we obtain the decomposition of $K_{43}^{(3)}$ into 1763 7-cycles.

\[\square\]

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REFERENCES


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Decomposing complete 3-uniform hypergraph $K^{(3)}_n$ into 7-cycles

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