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DECOMPOSING COMPLETE 3-UNIFORM HYPERGRAPH $K_n^{(3)}$ INTO 7-CYCLES

Meihua, Meiling Guan, Jirimutu

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Abstract. We use the Katona–Kierstead definition of a Hamiltonian cycle in a uniform hypergraph. A decomposition of complete k-uniform hypergraph $K_n^{(k)}$ into Hamiltonian cycles was studied by Bailey–Stevens and Meszka–Rosa. For $n \equiv 2, 4, 5 \pmod 6$, we design an algorithm for decomposing the complete 3-uniform hypergraphs into Hamiltonian cycles by using the method of edge-partition. A decomposition of $K_n^{(3)}$ into 5-cycles has been presented for all admissible $n \leq 17$, and for all $n = 4^m + 1$ when m is a positive integer. In general, the existence of a decomposition into 5-cycles remains open. In this paper, we show if $42 \mid (n-1)(n-2)$ and if there exist $\lambda = \frac{(n-1)(n-2)}{42}$ sequences $(k_{i_0}, k_{i_1}, \ldots, k_{i_6})$ on $D_{all}(n)$, then $K_n^{(3)}$ can be decomposed into 7-cycles. We use the method of edge-partition and cycle sequence. We find a decomposition of $K_{37}^{(3)}$ and $K_{43}^{(3)}$ into 7-cycles.

Keywords: uniform hypergraph, 7-cycle, cycle decomposition.

Mathematics Subject Classification: 05C65, 05C85.

1. INTRODUCTION

The notion of hamiltonicity in hypergaphs was introduced by Berge in 1970. A problem on decomposing complete 3-uniform hypergraphs into Hamilton cycles has been completely solved by Verall in 1994 [12]. It is also worth to mention a definition of a cycle by Berge. Recently, in [5] there were given necessary and sufficent conditions for a decompostion of the complete 3-uniform hypergraph of order n into 4-cycles. Meanwhile, many papers studied the two different definitions of a Hamiltonian cycle in [6,13], which are due to Katona and Kierstead, Wang and Lee, respectively. In fact, the two different definitions of a Hamiltonian cycle are the same. A decomposition of complete k-uniform hypergraphs into Hamiltonian cycles has been considered in [1, 3, 4, 6, 10, 13–15]. In the paper [10], Hamiltonian decompositions of $K_n^{(3)}$ for all admissible $n \leq 32$ has been resolved. Recently, by programming, using the method of edge-partition and cycle sequence, we have obtained some results for all admissible $32 < n \leq 46$ and $n \neq 43$ in [2]. Meszka and Rosa have introduced a necessary condition for the existence of 5-cycles such a decomposition is that $n \equiv$1,2,5,7,10 or 11(mod 15). In [10], the problem of decomposing the complete 3-uniform hypergraph into 5-cycles and $\ell(\geq 5)$-cycles are open. A decomposition of $K_n^{(3)}$ into 5-cycles exists for all admissible $n \leq 17$, for all $n = 4^m + 1$, m a positive integer. In [7,8], the author proposed to find a decomposition of $K_n^{(3)}$ into 5-cycles for $n \in \{5, 7, 10, 11, 16, 17, 20, 22, 26\}$ and shown that if $K_n^{(3)}$ can be decomposed into 5-cycles, then $K_{5n}^{(3)}$ can also be decomposed into 5-cycles. In [9], we found a decomposition of $K_n^{(3)}$ into 7-cycles for $n \in \{7, 8, 14, 16, 22, 23\}$ and have shown if $K_n^{(3)}$ can be decomposed into 7-cycles, then $K_{7n}^{(3)}$ can also be decomposed into 7-cycles. However, the problem of decomposing the complete 3-uniform hypergraph into 7-cycles are difficult. In this paper, under some conditions, we show if $42 \mid (n-1)(n-2)$ and if there exist $\lambda = \frac{(n-1)(n-2)}{42}$ sequences $(k_{i_0}, k_{i_1}, \ldots, k_{i_6})$ on $D_{all}(n)$, then $K_n^{(3)}$can be decomposed into 7-cycles.

2. PRELIMINARIES

A hypergraph $H = (V, E)$ consists of a finite set V of vertices with a family E of subsets of V, called hyperedges (or simply edges). If each (hyper)edge has a size k, we say that H is a k-uniform hypergraph. In particular, the complete k-uniform hypergraph on n vertices has all k-subsets of $\mathbb{Z}_n = \{0, 1, \ldots, n-1\}$ as edges, denote it by $K_n^{(k)}$. The set of (hyper)edges of $K_n^{(3)}$ is denoted by $\varepsilon(K_n^{(3)})$.

Definition 2.1. Let $H = (V, E)$ be a k-uniform hypergraph. An *ℓ-cycle* in H is a cyclic ordering $(v_0, v_1, \ldots, v_{\ell-1})$ of the elements of V such that each mod k consecutive k-tuple of vertices is an edge of H, where $3 \leq k < \ell - 1$.

Definition 2.2. An *ℓ-cycle decomposition* of H is a partition of the set of (hyper)edges of H into mutually-edge-disjoint ℓ-cycles.

We introduce the method of edge-partition and the cycle-model from [3].

Definition 2.3 ([1])**.** Let $T = \{a, b, c\}$ be a triple of distinct elements of $\mathbb{Z}_n$. Then its *difference pattern* $\pi(T)$ is the equivalence class of ordered triples containing cyclic rotations of $(b-a, c-b, a-c)$ and $(c-a, b-c, a-b)$ (where the differences are taken modulo n).

Clearly, the three differences sum to zero. Therefore, if we know that the first two differences are x and y, then the third is $n - x - y$. Omitting the third number, we obtain a difference pair. Using edge-partition of $K_n^{(3)}$ as in paper [3], all difference pairs of the hypergraph $K_n^{(3)}$ may be obtained. Let $\mathbb{Z}$ be the set of integers, n be a fixed positive integer, and $\mathbb{Z}_n = \{0, 1, \ldots, n-1\}$. Let

$$D_{all}(n) = \{(k_1, k_2) \mid 1 \leq k_1, k_2 \leq n-1, \text{ and } k_1 + k_2 \neq n\},$$
$$D(n) = D_e \cup D_l \cup D_m,$$

where

$$D_e = \Big\{(k_1, k_2) \in D_{all}(n) \mid k_1 = k_2 = k, 1 \le k < \frac{n}{2}\Big\},$$
$$D_l = \Big\{(k_1, k_2) \in D_{all}(n) \mid 1 \le k_1 < k_2 < \frac{n-k_1}{2}\Big\},$$
$$D_m = \Big\{(k_2, k_1) \in D_{all}(n) \mid (k_1, k_2) \in D_l\Big\}.$$

Given a difference pair $(k_1, k_2) \in D_{all}(n)$ and an integer $m \in \mathbb{Z}_n$, define a subhypergraph of $K_n^{(3)}$ generated by (k_1, k_2) as follows:

$$E(m; k_1, k_2) = \{m,\ m + k_1,\ m + k_1 + k_2\} \pmod{n},$$

We introduce the notation

$$H(k_1, k_2) = \{E(m; k_1, k_2) \mid m \in \mathbb{Z}_n\},$$

where the addition is performed modulo n.

Now we repeat some of the results from [3] to make the paper self-contained.

Lemma 2.4 ([3])**.** *Let (k_1, k_2) and (k_1', k_2') be arbitrary two distinct difference pairs in $D_{all}(n)$ we have either*

$$H(k_1, k_2) \cap H(k_1', k_2') = \emptyset$$

or

$$H(k_1, k_2) = H(k_1', k_2')$$

and a necessary and sufficient condition for the second equation is

$$(k_1, k_2) \equiv \begin{cases} (k_1', k_2') & \textit{or} \\ (k_1' + k_2', -k_2') & \textit{or} \\ (-k_1', k_1' + k_2') & \textit{or} \\ (k_2', -k_1' - k_2') & \textit{or} \\ (-k_1' - k_2', k_1') & \textit{or} \\ (-k_2', -k_1') \end{cases} \pmod{n}.$$

Definition 2.5 ([3])**.** Let (k_1, k_2) and (k_1', k_2') be arbitrary two distinct difference pairs in $D_{all}(n)$. We say (k_1, k_2) and (k_1', k_2') are equivalent if $H(k_1, k_2) = H(k_1', k_2')$. This is denoted by $(k_1', k_2') \sim (k_1, k_2)$.

Lemma 2.6 (Edge-partition of $K_n^{(3)}$, [3])**.** *For any $K_n^{(3)}$,*

$$\varepsilon(K_n^{(3)}) = \bigcup_{(k_1, k_2) \in D(n)} H(k_1, k_2),$$

where $(k_1, k_2) \in D(n)$. If $k_1 \ne k_2$, for convenience, we use (k_1, k_2) to denote (k_1, k_2) and (k_2, k_1).

Definition 2.7. Let n be a positive integer. For any $0 \le i,\ j \le \ell - 1,\ (k_i, k_{i+1}) \in D_{all}(n)$, and $(k_i, k_{i+1}) \not\sim (k_j, k_{j+1})$ $(i \neq j)$, let $(k_0, k_1, \ldots, k_{\ell-1})$ be a sequence on $D_{all}(n)$. The sequence $(k_0, k_1, \ldots, k_{\ell-1})$ induces the cycle sequence

$$(r_0, r_1, \ldots, r_{\ell-1}) \tag{2.1}$$

Sequence (2.1) satisfies the following two conditions:

(a) $r_0 = 0, \sum_{i=0}^{j} k_i \equiv r_j \pmod{n},\ r_\ell = 0.$

(b) For any $i,\ j\ (i \neq j),\ r_i \neq r_j$.

Then $(r_0, r_1, \ldots, r_{\ell-1})$ is an ℓ-cycle, denoted by $C_\ell = (r_0, r_1, \ldots, r_{\ell-1})$, called *base cycle*. According to the definition of difference pattern $\pi(T)$, we obtain the set of ℓ-cycles $\{C_\ell + i \mid i \in \mathbb{Z}_n\}$, where $C_\ell + i = (r_0 + i, r_1 + i, \ldots, r_{\ell-1} + i) \pmod{n}$. In particular, if $\ell = n$, then $(r_0, r_1, \ldots, r_{n-1})$ is a base Hamiltonian cycle, denoted by $C_n = (r_0, r_1, \ldots, r_{n-1})$.

Definition 2.8. Let n be a positive integer, for any $0 \le i,\ j \le \ell - 1,\ (k_i, k_{i+1}) \in D_{all}(n)$, and $(k_i, k_{i+1}) \not\sim (k_j, k_{j+1})$ $(i \neq j)$. Given a sequence $(k_0, k_1, \ldots, k_{\ell-1})$ in $D_{all}(n)$ and an integer $m \in \mathbb{Z}_n$, define a subhypergraph of $K_n^{(3)}$ generated by $(k_0, k_1, \ldots, k_{\ell-1})$ as follows:

$$(m; k_0, k_1, \ldots, k_{\ell-1}) = \Big\{m,\ m + k_0,\ m + k_0 + k_1, \ldots,\ \sum_{j=0}^{\ell-1} k_j\Big\} \pmod{n},$$

$$E(m; k_0, k_1, \ldots, k_{\ell-1}) = \Big\{m + \sum_{j=0}^{i-1} k_j, m + \sum_{j=0}^{i} k_j, m + \sum_{j=0}^{i+1} k_j\Big\} \pmod{n},$$

where $k_\ell = k_0$.

We introduce the notation

$$H(k_0, k_1, \ldots, k_{\ell-1}) = \{E(m; k_0, k_1, \ldots, k_{\ell-1}) \mid m \in \mathbb{Z}_n\},$$

where the addition is performed modulo n.

Lemma 2.9. *Let n be a positive integer, for any $0 \le i, j \le \ell - 1, (k_i, k_{i+1}) \in D_{all}(n)$, and $(k_i, k_{i+1}) \not\sim (k_j, k_{j+1})$ $(i \neq j)$. If $(k_0, k_1, \ldots, k_{\ell-1})$ is a sequence on $D_{all}(n)$, then*

$$H(k_0, k_1, \ldots, k_{\ell-1}) = \bigcup_{i=0}^{\ell-1} H(k_i, k_{i+1}),$$

where $k_\ell = k_0$.

Proof. For any $0 \le i \le \ell - 1$ and an integer $m \in \mathbb{Z}_n$, there is the edge of the $H(k_0, k_1, \ldots, k_{\ell-1})$ as follows:

$$\Big\{m + \sum_{j=0}^{i-1} k_j, m + \sum_{j=0}^{i} k_j, m + \sum_{j=0}^{i+1} k_j\Big\} \pmod{n}.$$

Obviously, the edge $\left\{m+\sum_{j=0}^{i-1} k_j, m+\sum_{j=0}^{i} k_j, m+\sum_{j=0}^{i+1} k_j\right\} \pmod n$ induces the difference pair (k_i, k_{i+1}), that is

$$H(k_i, k_{i+1}) = \left\{m + \sum_{j=0}^{i-1} k_j, m + \sum_{j=0}^{i} k_j, m + \sum_{j=0}^{i+1} k_j\right\} \pmod n.$$

Hence, we have

$$H(k_0, k_1, \ldots, k_{\ell-1}) = \bigcup_{i=0}^{\ell-1} H(k_i, k_{i+1}). \qquad \square$$

3. DECOMPOSING $K_n^{(3)}$ INTO 7-CYCLES

Theorem 3.1. *A necessary condition for the decomposition of complete 3-uniform hypergraph $K_n^{(3)}$ into 7-cycles is $42 \mid n(n-1)(n-2)$.*

Theorem 3.2. *If there exist $\lambda = \frac{(n-1)(n-2)}{42}$ sequences $(k_{i_0}, k_{i_1}, \ldots, k_{i_6})$ on $D_{all}(n)$ by Definition 2.5, then the sequences $(k_{i_0}, k_{i_1}, \ldots, k_{i_6})$ satisfy the following condition: if $(k_{i_\alpha}, k_{i_{(\alpha+1)}}) \not\sim (k_{j_\beta}, k_{j_{(\beta+1)}})$ $(i \neq j, \alpha \neq \beta, i, j \in [1, \lambda], \alpha, \beta \in [0, 5])$, then $K_n^{(3)}$ can be decomposed into 7-cycles.*

Proof. Let there exist $\lambda = \frac{(n-1)(n-2)}{42}$ sequences $(k_{i_0}, k_{i_1}, \ldots, k_{i_6})$ on $D_{all}(n)$ by Definition 2.5. The sequence $(k_{i_0}, k_{i_1}, \ldots, k_{i_6})$ induces the cycle sequence

$$(r_{i_0}, r_{i_1}, \ldots, r_{i_6}), \quad 1 \leq i \leq \lambda. \tag{3.1}$$

Sequence (3.1) satisfies the following two conditions:

(a) $r_{i_0} = 0$, $\sum_{\alpha=0}^{j} k_{i_\alpha} \equiv r_{i_j} \pmod n$, $r_{i_7} = 0$.
(b) For any $\alpha, \beta \in [0, 6]$ $(\alpha \neq \beta)$, $r_{i_\alpha} \neq r_{i_\beta}$.

Obviously, we obtain the set of base cycle λ 7 cycles. We can decompose the edges of $K_n^{(3)}$ into $n\lambda$ 7-cycles produced by λ-base 7-cycles. By the structure of base cycle, we obtain the set of 7-cycles $\{C_{7_\alpha} + i \mid i \in \mathbb{Z}_n, \alpha \in [1, \lambda]\}$. By the method of edge-partition, we obtain a decomposition of $K_n^{(3)}$ into $n\lambda$ 7-cycles, that is,

$$\begin{aligned}\varepsilon(K_n^{(3)}) &= \bigcup_{(k_1,k_2)\in(D(n)} H(k_1, k_2) = \bigcup_{(k_{i_0},k_{i_1},\ldots,k_{i_6})\in D_{all}(n)} H(k_{i_0}, k_{i_1}, \ldots, k_{i_6})\\ &= \bigcup_{i=1}^{\lambda} \{C_{7_i} + j,\ j \in \mathbb{Z}_n\}.\end{aligned}$$

Hence, we obtain a decomposition of $K_n^{(3)}$ into $n\lambda$ 7- cycles. $\square$

Example 3.3. *$K_{37}^{(3)}$ can be decomposed into 7-cycles.*

Proof. We can decompose the edges of $K_{37}^{(3)}$ into 1110 7-cycles produced by 30 base 7-cycles as follows. According to our method, we have

$$|\varepsilon(K_{37}^{(3)})| = \binom{37}{3} = 7770$$

edges and $7 \mid 7770$. We have

$$\begin{aligned} D(37) = \{&(1,1),(2,2),(3,3),(4,4),\ (5,5),\ (6,6),(7,7),(8,8),(9,9),\ (10,10),\ (11,11),\\ &(12,12),(13,13),(14,14),(15,15),\ (16,16),\ (17,17),\ (18,18),\ (1,2),\ (1,3),\\ &(1,4),(1,5),(1,6),(1,7),\ (1,8),(1,9),(1,10),(1,11),\ (1,12),(1,13),\ (1,14),\\ &(1,15),(1,16),\ (1,17),\ (2,3),\ (2,4),(2,5),(2,6),\ (2,7),(2,8),(2,9),\ (2,10),\\ &(2,11),(2,12),\ (2,13),\ (2,14),\ (2,15),\ (2,16),\ (2,17),\ (3,4),\ (3,5),\ (3,6),\\ &(3,7),\ (3,8),\ (3,9),\ (3,10),\ (3,11),\ (3,12),\ (3,13),\ (3,14),(3,15),\ (3,16),\\ &(4,5),(4,6),(4,7),(4,8),(4,9),(4,10),(4,11),(4,12),(4,13),\ (4,14),\ (4,15),\\ &(4,16),\ (5,6),\ (5,7),\ (5,8),\ \ (5,9),\ (5,10),\ (5,11),\ (5,12),\ (5,13),\ (5,14),\\ &(5,15),\ (6,7),\ (6,8),\ \ (6,9),\ (6,10),\ (6,11),\ (6,12),\ (6,13),(6,14),\ (6,15),\\ &(7,8),(7,9),(7,10),(7,11),(7,12),(7,13),(7,14),(8,9),\ (8,9),(8,10),(8,11),\\ &(8,12),(8,13),(8,14),(9,10),(9,11),(9,12),(9,13),(10,11),(10,12),(10,13),\\ &(11,12)\}. \end{aligned}$$

Now, we find the decomposition of $K_{37}^{(3)}$. On $D(37)$, according to Definition 2.5, we obtain $\lambda = 30$ sequences as follows:

(1) $(1,1,2,1,3,1,28)$, (2) $(1,4,35,5,31,5,30)$, (3) $(1,7,33,6,8,2,25)$,
(4) $(1,9,30,4,1,10,27)$, (5) $(1,12,27,2,9,1,22)$, (6) $(1,13,26,3,2,12,17)$,
(7) $(1,15,24,3,3,12,16)$, (8) $(1,16,23,3,5,8,18)$, (9) $(1,21,18,2,15,4,13)$,
(10) $(1,24,15,2,16,5,11)$, (11) $(1,26,14,4,29,31,7)$, (12) $(2,18,22,16,6,16,14)$,
(13) $(2,23,17,3,6,14,9)$, (14) $(2,28,12,28,30,5,6)$, (15) $(3,7,3,26,4,10,21)$,
(16) $(3,14,28,11,9,10,16)$, (17) $(3,16,25,31,10,11,15)$, (18) $(3,21,20,18,11,13,12)$,
(19) $(3,23,18,32,8,17,20)$, (20) $(4,5,4,25,4,21,11)$, (21) $(4,16,6,22,4,23,9)$,
(22) $(4,9,7,18,5,24,7)$, (23) $(4,14,30,27,24,32,5)$, (24) $(5,5,7,22,5,15,15)$,
(25) $(5,18,26,28,17,7,10)$, (26) $(5,20,28,27,18,6,7)$, (27) $(6,11,12,9,15,8,13)$,
(28) $(6,19,26,17,13,14,16)$, (29) $(6,21,8,9,9,13,8)$, (30) $(7,19,25,27,25,23,14)$,

Let D be a collection of 30 sequences above. Thus, they correspond to 30-base 7-cycles:

$$\begin{aligned} C_{7(1)} &= (0,1,2,4,5,8,9), & C_{7(2)} &= (0,1,5,3,8,2,7),\\ C_{7(3)} &= (0,1,8,4,2,10,12), & C_{7(4)} &= (0,1,10,3,7,8,18),\\ C_{7(5)} &= (0,1,13,3,5,14,15), & C_{7(6)} &= (0,1,14,3,6,8,20),\\ C_{7(7)} &= (0,1,16,3,6,9,21), & C_{7(8)} &= (0,1,17,3,6,11,19),\\ C_{7(9)} &= (0,1,22,3,5,20,24), & C_{7(10)} &= (0,1,25,3,5,21,26),\\ C_{7(11)} &= (0,1,27,3,7,36,30), & C_{7(12)} &= (0,2,20,5,1,7,23),\\ C_{7(13)} &= (0,2,25,5,814,28), & C_{7(14)} &= (0,2,30,5,33,26,31),\\ C_{7(15)} &= (0,3,10,13,2,6,16), & C_{7(16)} &= (0,3,17,8,2,11,21),\\ C_{7(17)} &= (0,3,19,7,1,11,22), & C_{7(18)} &= (0,3,24,7,1,12,25),\\ C_{7(19)} &= (0,3,26,7,2,10,27), & C_{7(20)} &= (0,4,9,13,1,5,26),\\ C_{7(21)} &= (0,4,10,16,1,5,28), & C_{7(22)} &= (0,4,13,20,1,6,30),\\ C_{7(23)} &= (0,4,18,11,1,25,20), & C_{7(24)} &= (0,5,10,17,2,7,22),\\ C_{7(25)} &= (0,5,23,12,3,20,27), & C_{7(26)} &= (0,5,25,16,6,24,30),\\ C_{7(27)} &= (0,6,17,29,1,16,24), & C_{7(28)} &= (0,6,25,14,31,7,21),\\ C_{7(29)} &= (0,6,27,35,7,16,29), & C_{7(30)} &= (0,7,26,14,4,29,15), \end{aligned}$$

By the method of edge-partition, we obtain the decomposition of $K_{37}^{(3)}$ into 1110 7-cycles, that is

$$\begin{aligned} \varepsilon(K_{37}^{(3)}) &= \bigcup_{(k_1,k_2)\in D(37)} H(k_1,k_2)\\ &= \bigcup_{(k_0,k_1,\ldots,k_6)\in D_{all}(37)} H(k_0,k_1,\ldots,k_6)\\ &= \bigcup_{i=1}^{30} \{C_{7(i)}+j,\ j\in\mathbb{Z}_{37}\}. \end{aligned}$$

Hence, we obtain the decomposition of $K_{37}^{(3)}$ into 1110 7-cycles. □

Example 3.4. $K_{43}^{(3)}$ *can be decomposed into 7-cycles.*

Proof. We can decompose the edges of $K_{43}^{(3)}$ into 1763 7-cycles produced by 41 base 7-cycles as follows. According to our method, we have

$$|\varepsilon(K_{43}^{(3)})| = \binom{43}{3} = 12341$$

edges and $7 \mid 12341$.

$D(43)=\{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6),(7,7),(8,8),(9,9),(10,10),(11,11),$
$(12,12),(13,13),(14,14),(15,15),(16,16),(17,17),(18,18),(19,19),$
$(20,20),(21,21),(1,2),(1,3),(1,4),(1,5),(1,6),(1,7),(1,8),(1,9),(1,10),$
$(1,11),(1,12),(1,13),(1,14),(1,6),(1,7),(1,8),(1,9),(1,10),(1,11),(1,12),$
$(1,13),(1,14),(1,15),(1,16),(1,15),(1,16),(1,17),(1,18),(1,19),(1,20),$
$(1,21),(2,3),(2,4),(2,5),(2,6),(2,7),(2,8),(2,9),(2,10),(2,11),(2,12)$
$(2,13),(2,14),(2,15),(2,16),(2,17),(2,18),(2,19),(2,20),(3,4),(3,5),$
$(3,6),(3,7),(3,8),(3,9),(3,10),(3,11),(3,12),(3,13),(3,14),(3,15),(3,16),$
$(3,17),(3,18),(3,19),(4,5),(4,6),(4,7),(4,8),(4,9),(4,10)(4,11),(4,12),$
$(4,13),(4,14),(4,15),(4,16),(4,17),(4,18),(4,19),(5,6),(5,7),(5,8),(5,9),$
$(5,10),(5,11),(5,12),(5,13),(5,14),(5,15),(5,16),(5,17),(5,18),(6,7),$
$(6,8),(6,9),(6,10),(6,11),(6,12),(6,13),(6,14),(6,15),(6,16),(6,17),$
$(6,18),(7,8),(7,9),(7,10),(7,11),(7,12),(7,13),(7,14),(7,16),(7,17),$
$(8,9),(8,10),(8,11)(8,12),(8,13),(8,14)(8,15),(8,16),(8,17),(9,10),$
$(9,11),(9,12),(9,13)(9,14),(9,15),(9,16),(10,11),(10,12),(10,13),(10,14),$
$(10,15),(10,16),(11,12),(11,13),(11,14),(11,15)\}.$

Now, we need to find the decomposition of $K_{43}^{(3)}$. On $D(43)$, according to Definition 2.5, we obtain 41 sequences as follows:

(1) $(1,1,2,1,3,1,34)$, (2) $(1,4,41,5,37,5,36)$,
(3) $(1,7,39,41,8,2,31)$, (4) $(1,9,36,4,1,10,25)$,
(5) $(1,12,33,2,9,1,28)$, (6) $(1,13,32,3,2,12,23)$,
(7) $(1,15,30,3,3,12,22)$, (8) $(1,16,29,3,5,8,24)$,
(9) $(1,21,24,2,15,3,20)$, (10) $(1,23,22,3,6,13,18)$,
(11) $(1,25,20,4,2,18,16)$, (12) $(1,27,18,3,7,17,13)$,
(13) $(1,30,15,2,16,11,11)$, (14) $(1,32,13,4,35,37,7)$,
(15) $(2,20,26,39,9,14,19)$, (16) $(2,24,22,39,10,18,14)$,
(17) $(22,29,17,3,9,20,6)$, (18) $(2,32,14,4,32,38,7)$,
(19) $(3,13,24,37,9,7,26)$, (20) $(3,15,32,37,10,19,13)$,
(21) $(3,16,31,7,11,10,21)$, (22) $(3,24,23,37,12,20,10)$,
(23) $(3,28,19,25,13,21,8)$, (24) $(3,31,16,38,8,26,7)$,
(25) $(4,7,5,28,5,16,23)$, (26) $(4,8,4,28,7,6,29)$,
(27) $(4,13,38,8,5,18,17)$, (28) $(4,14,34,35,15,6,21)$,
(29) $(4,15,37,7,17,9,16)$, (30) $(4,29,19,36,20,12,9)$,
(31) $(5,5,6,10,23,15,22)$, (32) $(5,7,7,25,5,19,18)$,
(33) $(5,12,8,19,6,23,13)$, (34) $(5,15,7,18,8,16,17)$,
(35) $(5,17,7,16,35,33,16)$, (36) $(6,16,15,8,22,8,11)$,
(37) $(7,15,20,13,16,34,24)$, (38) $(7,20,34,17,27,29,22)$,
(39) $(7,22,38,29,28,30,25)$, (40) $(9,9,11,15,18,11,13)$,
(41) $(9,10,11,17,12,12,15)$.

Let D be a collection of the 41 sequences above. Thus they correspond to 41 base 7-cycles:

$$
\begin{aligned}
C_{7(1)} &= (0,1,2,4,5,8,9), & C_{7(2)} &= (0,1,5,3,8,2,7),\\
C_{7(3)} &= (0,1,8,4,2,10,12), & C_{7(4)} &= (0,1,,10,3,7,8,18),\\
C_{7(5)} &= (0,1,13,3,5,14,15), & C_{7(6)} &= (0,1,14,3,6,8,20),\\
C_{7(7)} &= (0,1,16,3,6,9,21), & C_{7(8)} &= (0,1,17,3,6,11,19),\\
C_{7(9)} &= (0,1,22,3,5,20,23), & C_{7(10)} &= (0,1,24,3,6,12,25),\\
C_{7(11)} &= (0,1,26,3,7,9,2), & C_{7(12)} &= (0,1,28,3,6,13,30),\\
C_{7(13)} &= (0,1,31,3,5,21,32), & C_{7(14)} &= (0,1,33,3,7,42,36),\\
C_{7(15)} &= (0,2,22,5,1,10,24), & C_{7(16)} &= (0,2,26,5,1,11,29),\\
C_{7(17)} &= (0,2,31,5,8,17,37), & C_{7(18)} &= (0,2,34,5,9,41,36),\\
C_{7(19)} &= (0,3,16,7,1,10,17), & C_{7(20)} &= (0,3,18,7,1,11,30),\\
C_{7(21)} &= (0,3,19,7,1,12,22), & C_{7(22)} &= (0,3,27,7,1,13,33),\\
C_{7(23)} &= (0,3,31,7,1,14,35), & C_{7(24)} &= (0,3,34,7,2,10,36),\\
C_{7(25)} &= (0,4,11,16,1,6,20), & C_{7(26)} &= (0,4,12,16,1,8,14),\\
C_{7(27)} &= (0,4,17,12,3,8,26), & C_{7(28)} &= (0,4,18,9,1,16,22),\\
C_{7(29)} &= (0,4,19,13,1,18,27), & C_{7(30)} &= (0,4,33,9,2,22,34),\\
C_{7(31)} &= (0,5,10,16,26,6,21), & C_{7(32)} &= (0,5,12,19,1,6,25),\\
C_{7(33)} &= (0,5,17,25,1,7,30), & C_{7(34)} &= (0,5,20,27,2,10,26),\\
C_{7(35)} &= (0,5,22,29,2,37,27), & C_{7(36)} &= (0,6,22,37,2,24,32),\\
C_{7(37)} &= (0,7,22,42,12,28,19), & C_{7(38)} &= (0,7,27,18,8,35,21),\\
C_{7(39)} &= (0,7,29,17,3,31,18), & C_{7(40)} &= (0,9,18,29,1,19,30),\\
C_{7(41)} &= (0,9,19,30,4,16,28).
\end{aligned}
$$

By the method of edge-partition, we obtain the decomposition of $K_{43}^{(3)}$ into 1763 7-cycles, that is,

$$
\begin{aligned}
\varepsilon(K_{43}^{(3)}) &= \bigcup_{(k_1,k_2)\in D(43)} H(k_1,k_2) = \bigcup_{(k_0,k_1,\ldots,k_6)\in D_{all}(43)} H(k_0,k_1,\ldots,k_6)\\
&= \bigcup_{i=1}^{41} \big\{C_{7(i)}+j,\ j\in \mathbb{Z}_{43}\big\}.
\end{aligned}
$$

Hence we obtain the decomposition of $K_{43}^{(3)}$ into 1763 7-cycles. □

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REFERENCES

[1] R. Bailey, B. Stevens, *Hamiltonian decompositions of complete k-uniform hypergraphs*, Discrete Math. **310** (2010), 3088–3095.

[2] H. Huo, L.Q. Zhao, W. Feng, Y.S. Yang, Jirimutu, *Decomposing the complete 3-uniform hypergraph $K_n^{(3)}$ into Hamiltonian cycles*, Journal of Mathematics **58** (2015), 965–976.

[3] Jirimutu, J. Wang, *Study on graph labelings and hypergraph decomposition*, Doctoral Dissertation, Dalian University of Technology, 2006.

[4] Jirimutu, J.F. Wang, *Hamiltonian decompositions of complete bipartite hypergraphs*, Acta Math. Appl. Sin. **4** (2001), 563–566.

[5] H. Jordon, G. Newkirk, *4-cycle decompositions of complete 3-uniform hypergraphs*, Australas. J. Combin. **71** (2017), 312–323.

[6] G.Y. Katona, H.A. Kierstead, *Hamiltonian chains in hypergraphs*, J. Graph Theory **30** (1999), 205–212.

[7] G.R. Li, Y.M. Lei, L.Q. Zhao, Y.S. Yang, Jirimutu, *Decomposing the complete 3-uniform hypergraph $K_n^{(3)}$ into 5-cycles*, Applied Mechanics and Materials **672–674** (2014), 1935–1939.

[8] G.R. Li, Y.M. Lei, L.Q. Zhao, Y.S. Yang, Jirimutu, *Decomposing the complete 3-uniform hypergraph $K_n^{(3)}$ into 5-cycles*, J. Math. Res. Appl. **36** (2016), 9–14.

[9] G.R. Li, Y.M. Lei, Y.S. Yang, Jirimutu, *Decomposing the complete 3-uniform hypergraph $K_n^{(3)}$ into 7-cycles* (2014), submitted.

[10] M. Meszka, A. Rosa, *Decomposing complete 3-uniform hypergraph into Hamiltonian cycles*, Australas. J. Combin. **45** (2009), 291–302.

[11] M. Meszka, A. Rosa, *A possible analogue of ρ-labellings for 3-uniform hypergraghs*, Electron. Notes Discrete Math. **60** (2017), 33–37.

[12] H. Verrall, *Hamiltonian decompositions of complete 3-uniform hypergraphs*, Discrete Math. **132** (1994), 333–348.

[13] J.F. Wang, Tony T. Lee, *Paths and Cycles of Hypergraphs*, Science in China (A) **42** (1999), 1–12.

[14] J.F. Wang, G.Y. Yan, *On cycle structure of hypergraph*, Chinese Science Bulletin **19** (2001), 1585–1589.

[15] B.G. Xu, J.F. Wang, *On the Hamiltonian decompositions of complete 3-uniform hypergraphs*, Electron. Notes Discrete Math. **11** (2002), 722–733.

Meihua

Mongolia University for the Nationalities
College of Mathematics of Inner
Tongliao, China 028043

Meiling Guan

Mongolia University for the Nationalities
College of Mathematics of Inner
Tongliao, China 028043

Jirimutu
jrmt@sina.com

Mongolia University for the Nationalities
College of Mathematics of Inner
Tongliao, China 028043

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