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ANTI-RAMSEY NUMBERS FOR DISJOINT COPIES OF GRAPHS

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Abstract. A subgraph of an edge-colored graph is called *rainbow* if all of its edges have different colors. For a graph G and a positive integer n, the *anti-Ramsey number* $ar(n,G)$ is the maximum number of colors in an edge-coloring of K_n with no rainbow copy of H. Anti-Ramsey numbers were introduced by Erdős, Simonovits and Sós and studied in numerous papers. Let G be a graph with anti-Ramsey number $ar(n,G)$. In this paper we show the lower bound for $ar(n,pG)$, where pG denotes p vertex-disjoint copies of G. Moreover, we prove that in some special cases this bound is sharp.

Keywords: anti-Ramsey number, rainbow number, disjoint copies.

Mathematics Subject Classification: 05C55, 05C15.

1. INTRODUCTION

A subgraph of an edge-colored graph is called *rainbow* if all of its edges have different colors. For a graph G and a positive integer n, the *anti-Ramsey number* $ar(n,G)$ is the maximum number of colors in an edge-coloring of K_n with no rainbow copy of G. Anti-Ramsey numbers were introduced by Erdős *et al.* [4]. They showed that these are closely related to Turán numbers. Since then numerous results were established for a variety of graphs H, including among others cycles [1,11,13], matchings [5,9,17], trees [10,12] and cycles with an edge added [8,15]. The paper of Fujita, Magnant and Ozeki [6] presents the survey of results of that type.

In this paper we consider the following problem. Given a connected graph G, the anti-Ramsey number $ar(n,G)$, we ask what can be said about $ar(m,pG)$, where pG denotes p vertex-disjoint copies of G. We give the lower bound for this number and discuss the sharpness of it. As far as we know the only considered graphs of this type were matchings.

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2. PRELIMINARIES

Graphs considered below will always be simple. Throughout the paper we use the standard graph theory notation. For a graph G the order of G is denoted by $|G|$ and the size is denoted by $\|G\|$. K_n and pG stand for, respectively, the complete graph on n vertices and the disjoint union of p copies of a graph G. A degree of a vertex v in a graph G is denoted by $d_G(v)$ and by $N_G(v)$ and $N_G[v]$ its open and closed neigborhoods, respectively. For a graph G and its subgraph H by $G - H$ we mean a graph obtained from G by deleting all vertices of H with all incident edges. If $W \subseteq V(G)$, then $G[W]$ denotes the subgraph of G induced by W. For a set S, by $|S|$ we denote the cardinality of S.

Additionally, we introduce the following notation. $C(G)$ is a set of colors used on the edges of a graph G, $C(v)$ is a set of colors used on the edges incident to a vertex v and $c(e)$ denotes a color of the edge e. For a given coloring of the edges of K_n we choose exactly one edge in each color. A subgraph F such that $V(F) = V(K_n)$ induced by these edges we call a *selective subgraph*.

We will need the following theorems.

Theorem 2.1 ([4])**.** *$ar(m, K_3) = m - 1$ for $m \geq 3$.*

Theorem 2.2 ([16])**.** *If G is a graph with $n \geq 3$ vertices such that $\|G\| > \binom{n-1}{2} + 1$, then G has a Hamiltonian cycle.*

Theorem 2.3 ([13])**.** *If $m \geq k \geq 3$ and r is the reminder of the division m by $k-1$, then*

$$ar(m, C_k) = \left\lfloor \frac{m}{k-1} \right\rfloor \binom{k-1}{2} + \binom{r}{2} + \left\lceil \frac{m}{k-1} \right\rceil.$$

Theorem 2.4 ([11, 14])**.** *$ar(m, K_{1,3}) = \lfloor \frac{m}{2} \rfloor + 1$, $m \geq 4$.*

Theorem 2.5 ([11, 14])**.** *$ar(m, K_{1,4}) = m + 1$, $m \geq 5$.*

3. LOWER BOUND

Theorem 3.1. *Let G be an arbitrary connected graph on $n \geq 3$ vertices and $m \geq p|V(G)|$. Then*

$$ar(m, pG) \geq \max\left\{ \binom{pn-2}{2} + 1, ar(m-p+1, G) + (p-1)m - \binom{p}{2} \right\}.$$

Proof. We color the edges of K_m as follows. To obtain the first number we choose K_{pn-2} and color it rainbowly and we color the remaining edges with one extra color. In such a way we do not obtain any rainbow pG and use exactly $\binom{pn-2}{2} + 1$ colors.

To obtain the second number we choose K_{p-1} and color it rainbowly, then we color the edges of $K_m - K_{p-1}$ with next $ar(m - p + 1, G)$ colors without producing rainbow G and finally we color the remaining edges each with next distinct colors. In such a way we do not obtain any rainbow pG and use exactly

$$ar(m-p+1,G)+(p-1)(m-p+1)+\binom{p-1}{2} = ar(m-p+1,G)+(p-1)m-\binom{p}{2}$$

colors, so the theorem is proved. □

It is worth to pay attention to the fact that the lower bound from Theorem 3.1 is not appropriate for a matching. In this case assuming $G = K_2$ it is reasonable to put $ar(m, K_2) = 0$. But in the construction of the coloring we must not use 0 colors on the rest of the graph. Similarly the colorings are based on the fact that by adding one new color we do not produce any copy of G which is not true for $G = K_2$. That is why matchings need a different treating. It is done in [5, 9, 17] by using an appropriate Turán number.

From this point of view $G = P_3$ is the smallest graph to consider.

In our paper we are interested in selecting graphs for which this lower bound can be sharp. We do not focus on cases when

$$\max\left\{\binom{pn-2}{2}+1, ar(m-p+1,G)+(p-1)m-\binom{p}{2}\right\} = \binom{pn-2}{2}+1,$$

since this can happen only for finitely many values of m. It is so, as the first expression is a constant and the second one is at least linear in m.

We state the following conjecture.

Conjecture 3.2. *Let G be a connected graph on $n \geq 3$ vertices and $m \geq p|V(G)|$. Then*

$$ar(m,pG) = ar(m-p+1,G)+(p-1)m-\binom{p}{2}$$

if and only if G is a tree.

In the next paragraphs we give the reasons which motivated us to state such a conjecture.

3.1. DISJOINT PATHS

It is easy to see that $ar(m, P_3) = 1$. By Theorem 3.1, it can be obtained that $ar(m, 2P_3) \geq m$ for $m \geq 7$ and $ar(6, 2P_3) \geq 7$. The next theorem shows that this lower bound is sharp. The result was also achieved by Bialostocki, Gilboa and Roditty [2], but with a different method of the proof, so we put the theorem into the paper.

Theorem 3.3.

$$ar(m,2P_3) = \begin{cases} 7 & \textit{for } m = 6, \\ m & \textit{for } m \geq 7. \end{cases}$$

Proof. The lower bound for $m \geq 7$ results from Theorem 3.1. For $m = 6$ we color the edges of a subgraph K_4 with distinct colors and all remaining edges of K_6 with one extra color.

To show the upper bound we color the edges of a complete graph $K_m = K$ with $m + 1$ colors and assume that there is no rainbow $2P_3$. By Theorem 2.1, there is a rainbow triangle T with vertices $\{u, v, w\}$. Let $C_R = C(K) \setminus C(T)$ and $V(K - T) = \{x_1, x_2, \ldots, x_{m-3}\}$. Note that if there is an edge $e \in E(K - T)$ with $c(e) \in C_R$, then we obtain a rainbow $2P_3$ consisting of the edge e, an edge $e' \in E(K - T)$ incident to it and to edges from $E(T)$ of colors different from $c(e')$. A contradiction. Hence we can assume that all edges of colors from C_R are placed between T and $K - T$.

Since $|C_R| = m-2$, at least one vertex from T is joined to at least two vertices from $K - T$ with edges of distinct colors from C_R. Let u be this vertex, $c(ux_1), c(ux_2) \in C_R$, $c(ux_1) \neq c(ux_2)$ and $C'_R = C_R \setminus \{c(ux_1), c(ux_2)\}$.

Note that we can assume that for each $i \in \{3, \ldots, m-3\}$ we have that $c(x_i v) \notin C'_R$ and $c(x_i w) \notin C'_R$, since otherwise there would be a rainbow $2P_3$: $x_1 u x_2$, $x_j v w$ ($x_j w v$, respectively) for a certain $j \in \{3, \ldots, m-3\}$. Since $|C'_R| = m - 4$, there is an edge of color from C'_R between $\{x_1, x_2\}$ and $\{v, w\}$. Let $x_1 v$ be this edge. Similarly as above we obtain that $c(x_i u) \notin C'_R \setminus \{c(x_1 v)\}$ for each $i \in \{3, \ldots, m-3\}$, otherwise $x_2 u x_j$, $x_1 v w$ is a rainbow $2P_3$ for certain $j \in \{3, \ldots, m-3\}$. Now there are only two edges left ($x_2 v$ and $x_2 w$) which are allowed to be colored with colors from $C'_R \setminus \{c(x_1 v)\}$. But $|C'_R \setminus \{c(x_1 v)| = m - 3$. A contradiction for $m \geq 8$. For $m = 6, 7$, $|C'_R \setminus \{c(x_1 v)\}| = 2$, so surely $x_3 x_2 w$, $u x_1 v$ is a rainbow $2P_3$, remembering that $c(x_2 x_3) \in C(T)$. □

The next theorem deals with three copies of P_3. It is a special case of a more general the result obtained by Gilboa and Roditty [7], namely $ar(m, pP_3) = (p-1)(m - \frac{p}{2}) + 1$ for $m > 5p + 1$. By a different method of the proof, we managed to decrease the constraint for m from 16 to 12 for $p = 3$.

Theorem 3.4. $ar(m, 3P_3) = 2m - 2$ *for* $m > 12$.

Proof. The lower bound results from Theorem 3.1. To show the upper bound we color the edges of a complete graph $K_m = K$ with $2m - 1$ colors arbitrarily and assume that there is no rainbow $3P_3$.

Let F be a selective subgraph of K containing the longest cycle and l denote its length. Since $|V(F)| = m$ and $|E(F)| = 2m - 1$, such a selective subgraph can be chosen. Moreover, there are at most two vertex-disjoint cycles in F, since otherwise a rainbow $3P_3$ is in K.

Note that if $l \geq 9$, then obviously a rainbow $3P_3$ is contained in K. Moreover, by Theorem 2.3, $l \geq 5$. Therefore $l \in \{5, 6, 7, 8\}$. Let $\mathcal{C}_l$ be the subgraph of F being the longest cycle.

Let $F_l = F[V(\mathcal{C}_l)]$, $B = F - \mathcal{C}_l$, $R = \{vw : v \in V(\mathcal{C}_l), w \in V(B)\}$ and $N = \{w \in V(B) : \text{ there exists } v \in V(\mathcal{C}_l) \text{ such that } vw \in E(F)\}$. Note that

$$\|F\| = \|F_l\| + \|B\| + |R|.$$

Case 1. $l = 8$.

Observe that $|N| = 0$, since otherwise a rainbow $3P_3$ is in K.

We show that there is at most one edge in B. Suppose that there are vertices $x_1, x_2, x_3, x_4 \in V(B)$ such that $x_1x_2, x_3x_4 \in E(F)$. If $x_2 = x_3$, then $x_1x_2x_4$ is rainbow. If $x_2 \neq x_3$, then at least one of paths $A^1 = x_1x_2x_3$ or $A^2 = x_4x_3x_2$ is rainbow. Possibly deleting the edge with color $c(x_2x_3)$ in $\mathcal{C}_8$ we obtain a rainbow subgraph of $\mathcal{C}_8$ which contains $2P_3$. It contradicts the assumption that there is no $3P_3$ in F. Therefore, $\|B\| \leq 1$.

Now, let $e = xy$ be an edge in K such that $x \in V(\mathcal{C}_8)$ and $y \in V(B)$. Obviously, $e \notin E(F)$ and $c(e)$ is one of colors from $C(\mathcal{C}_8)$. Then $\|F_8\| \leq 23$. Otherwise, deleting the edge with color $c(e)$ in F_8, by Theorem 2.2 we obtain a rainbow hamiltonian graph of order 8 without a color $c(e)$ and joining e to the hamiltonian cycle we obtain a rainbow P_9 in K and hence a rainbow $3P_3$ is in K. Hence,

$$\|F\| = \|F_8\| + \|B\| \leq 23 + 1 = 24 < 2m - 1,$$

a contradiction.

Case 2. $l = 7$.

Observe that $|N| \leq 1$. Otherwise it is easy to obtain $3P_3$ in F.

Analogously as in previous cases, we can show that $\|B\| \leq 1$.

Suppose than that $|N| = 0$. So, $\|F_7\| \leq 21$. Hence,

$$\|F\| \leq 21 + 1 = 22 < 2m - 1,$$

a contradiction.

Assume that $|N| = 1$ and $N = \{x\}$. Similarly as in a previous case, by Theorem 2.2, we obtain that $F[V(\mathcal{C}_7) \cup \{x\}]$ contains at most 23 edges. Hence,

$$\|F\| \leq 23 + 1 = 24 < 2m - 1,$$

a contradiction.

Case 3. $l = 6$.

Analogously as in Case 1, we can show that $\|B\| \leq 1$.

Denote the consecutive vertices in $\mathcal{C}_6$ by $\{c_0, c_1, \ldots, c_5\}$ and by $d_R(x)$ the number of edges in R incident with x. Observe that since $\|B\| \leq 1$, we have

$$|R| \geq 2m - 1 - (15 + 1) = 2m - 17 \geq m - 4$$

for every $m > 12$. It implies that $|R| > 0$ and there are at least two distinct vertices c_i, c_j such that $d_R(c_i) \geq 1$ and $d_R(c_j) \geq 1$. Without loss of generality, we can assume that $d_R(c_0) \geq 2$.

The assumption that $d_R(c_k) \geq 1$ for certain $k \in \{1, 2, 4, 5\}$ leads us to a contradiction with the assumption that there is no $3P_3$ in F. Therefore c_3 is the other vertex with neighbors in N and moreover $d_R(c_3) \geq 2$.

If $|N| \leq 3$, then $|R| \leq 6$, a contradiction. So $|N| \geq 4$ which means that we can choose a rainbow $2P_3$ in F with middle vertices c_0 and c_3 and endpoints in N.

The rainbow $3P_3$ in K we can find as follows.

If $c(c_1c_4) \notin C(2P_3)$, then we are done, since at least one of the paths $c_5c_4c_1$, $c_2c_1c_4$ is rainbow in K.

So suppose that $c(c_1c_4) \in C(2P_3)$. Without loss of generality let xc_0y be one of above mentioned rainbow $2P_3$ and $c(c_1c_4) = c(xc_0)$. Then the other P_3 with middle vertex c_3, $c_2c_1c_4$ and yc_0c_5 form a rainbow $3P_3$ in K.

Hence we obtain a contradiction.

Case 4. $l = 5$.

Denote the consecutive vertices in $\mathcal{C}_5$ by $\{c_0, c_1, \ldots, c_4\}$.

Suppose that $P_3 = P$ is contained in B. Note that either one can find rainbow $2P_3$ with one vertex in $V(B) \setminus V(P)$ and five vertices in $V(C_5)$ or for each $u \in (V(B) \setminus V(P))$ we have $c(uc_i) = c(c_{i+2 \mod 5}c_{i+3 \mod 5})$. In the latter case we have a rainbow $2P_3$: $c_0u_1c_1$, $c_2u_2c_3$, where $u_1, u_2 \in (V(B) \setminus V(P))$. The rainbow $2P_3$ forms a rainbow $3P_3$ with P. A contradiction.

Therefore we can assume that $B = sK_2 \cup (m - 5 - 2s)K_1$.

Note that each vertex $u \in V(B)$ is adjacent to at most two vertices on the cycle (c_i, $c_{i+2 \mod 5}$) otherwise F contains a longer cycle.

Moreover, if at least one $u \in V(B)$ has two neighbors on the cycle, then $\|F_5\| \leq 8$ ($c_{i+1 \mod 5}c_{i+4 \mod 5} \notin E(F)$, $c_{i+1 \mod 5}c_{i+3 \mod 5} \notin E(F)$) otherwise F contains a longer cycle.

Finally, note that if $u_1u_2 \in E(F)$, then there are at most two edges between $\{u_1, u_2\}$ and $V(C_5)$ otherwise F contains a longer cycle.

Hence if at least one $u \in V(B)$ has two neighbors on the cycle, then

$$2m - 1 = \|F\| = \|F_5\| + \|B\| + |R| \leq 8 + s + 2s + 2[(m-5) - 2s] = 2m - s - 2.$$

A contradiction.

If all $u \in V(B)$ have at most one neighbor on the cycle, then

$$2m - 1 = \|F\| = \|F_5\| + \|B\| + |R| \leq 10 + s + (m-5) = m + s + 5.$$

Since $s \leq \lfloor \frac{m-5}{2} \rfloor$, we have a contradiction. □

Next we consider two copies of a star with three rays.

Theorem 3.5. *Let $m \geq 69$. Then $ar(m, 2K_{1,3}) = \lfloor \frac{m-1}{2} \rfloor + m$.*

Proof. The lower bound results from Theorems 3.1 and 2.4. To show the upper bound we color the edges of a complete graph $K_m = K$ with $\lfloor \frac{m-1}{2} \rfloor + m + 1$ colors arbitrarily and assume that there is no rainbow $2K_{1,3}$.

Let F be a selective subgraph of K chosen in such a way that the maximal degree $\Delta(F)$ is as big as possible and let x_0 be the vertex of K such that $d(x_0) = \Delta(F) = d$. Note that, since $\lfloor \frac{m-1}{2} \rfloor + m + 1 > m + 1 = ar(m, K_{1,4})$ (see Theorem 2.5), we can assume that $d \geq 4$. Obviously $d \leq m - 1$. Let $N_F(x_0) = \{x_1, x_2, \ldots, x_d\}$ and $V(F) \setminus N_F[x_0] = \{x_{d+1}, x_{d+2}, \ldots, x_{m-1}\}$. The latter set is empty if $d = m - 1$.

Let us consider the case $d \geq 8$ firstly. Let $F^- = F - x_0$. Note that

$$\|F^-\| \geq \|F\| - (m-1) = \left\lfloor \frac{m-1}{2} \right\rfloor + m + 1 - (m-1) = \left\lfloor \frac{m-1}{2} \right\rfloor + 2. \tag{3.1}$$

Since $|F^-| = m-1$, we obtain that $K_{1,2} \subset F^-$. Without loss of generality let $x_1x_2x_3$ be this star with the center x_1. Note that there is no other edges with the end x_1 in F^- otherwise there would be rainbow $2K_{1,3}$ in F (one star with center x_1 and one with x_0).

Moreover, note that G^- does not contain two edge-disjoint stars $K_{1,2}$. If $x_1x_2x_3$ and $x_ix_jx_k$ be such a stars with centers x_1 and x_i, respectively, then at least one of the stars $x_1x_2x_3x_i$ or $x_ix_jx_kx_1$ would be rainbow in K and form a rainbow $2K_{1,3}$ together with a certain star with a center x_0, even if $c(x_1x_i) \in C(x_0)$. Therefore G^- is a subset of (i) $K_{1,2} \cup \lfloor \frac{m-4}{2} \rfloor K_2$ or (ii) $P_4 \cup \lfloor \frac{m-5}{2} \rfloor K_2$ or (iii) $K_3 \cup \lfloor \frac{m-4}{2} \rfloor K_2$. In case (i) we have $\|F^-\| \leq 2 + \lfloor \frac{m-4}{2} \rfloor = \lfloor \frac{m}{2} \rfloor$ and in case (ii) $\|F^-\| \leq 3 + \lfloor \frac{m-5}{2} \rfloor = \lfloor \frac{m+1}{2} \rfloor$, which is a contradiction to (3.1). There is no similar contradicion in case (iii) only in case $F^- \simeq K_3 \cup \lfloor \frac{m-4}{2} \rfloor K_2$. In that case let x_1, x_2, x_3 be the vertices of the triangle and x_ix_j be an edge of F^-, $i,j \notin \{x_1,x_2,x_3\}$. Now we look at colors of the edges in K. If $c(x_1x_i) \notin \{c(x_1x_2), c(x_1x_3)\}$ or $c(x_2x_i) \notin \{c(x_2x_1), c(x_2x_3)\}$ or $c(x_3x_i) \notin \{c(x_3x_2), c(x_3x_1)\}$, then we have a rainbow star $K_{1,3}$ with a centrum x_s for a certain $s \in \{1,2,3\}$ which forms a rainbow $2K_{1,3}$ with the rainbow $K_{1,3}$ with a centrum x_0. Therefore we can assume that $c(x_1x_i) \in \{c(x_1x_2), c(x_1x_3)\}$ and $c(x_2x_i) \in \{c(x_2x_1), c(x_2x_3)\}$ and $c(x_3x_i) \in \{c(x_3x_2), c(x_3x_1)\}$. So there is a rainbow star $K_{1,3}$ $x_ix_sx_tx_j$ with a centrum x_i for a certain $s,t \in \{1,2,3\}$. So again we have the rainbow $2K_{1,3}$ with the rainbow $K_{1,3}$ with a centrum x_0. A contradiction.

Now consider the case $4 \leq d \leq 7$. Note that each x_i, $i = 1,2,\ldots,d$ can have at most two neighbors in $\{x_{d+1}, x_{d+2}, \ldots, x_{m-1}\}$ otherwise we can easily find $2K_{1,3}$ in F. So there is at most $d + 2d + \binom{d}{2}$ edges with at least one endpoint in $\{x_0, x_1, x_2, \ldots, x_d\}$. Hence at least $\lfloor \frac{m-1}{2} \rfloor + m + 1 - 3d - \binom{d}{2}$ edges have both endpoints in $\{x_{d+1}, x_{d+2}, \ldots, x_{m-1}\}$. Note that at least one of these vertices has three neighbors in this set, since

$$\left\lfloor \frac{m-1}{2} \right\rfloor + m + 1 - 3d - \binom{d}{2} > 2(m-d-1)/2$$

for $m \geq 69$. So again we have $2K_{1,3}$ in F. A contradiction. □

3.2. DISJOINT TRIANGLES

It is unlikely that the lower bound we discuss is sharp in any case. By the results of Erdős *et al.* [3,4], it follows that if G is a graph which is not bipartite and does not become bipartite after deleting a single edge, then $ar(m,G)$ and $ex(m,\mathcal{G}^-)$ are asymptotically equal, where $ex(m,\mathcal{H})$ denotes well known Turán number for a family $\mathcal{H}$ and $\mathcal{G}^-$ is the family of all graphs obtained from G by deleting one edge. Moreover, recently Schiermeyer and Sótak [18] showed that for a graph G with cyclomatic number at least 2 the anti-Ramsey number $ar(m,G)$ cannot be bounded above by a function which is linear in m.

As an example we present the following theorem.

Theorem 3.6. *Let $m \geq 6$. Then $ar(m, 2K_3) \geq \lfloor \frac{m^2}{4} \rfloor + 1$.*

Proof. To construct an appropriate coloring of the edges of K_m we proceed as follows. We choose a triangle-free subgraph H with maximum possible number of edges (Turán graph) and assign to each edge a different color. Then we put one extra color to all remaining edges. Certainly, by the Turán theorem, $|E(H)| = \lfloor \frac{m^2}{4} \rfloor$. Obviously, there is no rainbow $2K_3$ in such a coloring, hence the proof is completed. □

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