

ON SOME DYNAMICAL RECONSTRUCTION PROBLEMS FOR A NONLINEAR SYSTEM OF THE SECOND-ORDER

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Abstract. The problem of reconstruction of unknown characteristics of a nonlinear system is considered. Solution algorithms stable with respect to the informational noise and computational errors are specified. These algorithms are based on the method of auxiliary positionally controlled models.

Keywords: ordinary differential equations, reconstruction.

Mathematics Subject Classification: 34A55, 49N45.

1. INTRODUCTION

Problems of determination of input of system through equation's solutions are often called reconstruction (identification) problems. Therefore it is assumed that the input information (results of measurements of current phase states of a dynamical system) are forthcoming in the process. As to unknown parameters, they should be reconstructed in the process too. One of the methods of solving similar problems was suggested in [3]. This method was based on the ideas of the theory of ill-posed problems [7] and actually reduces the identification problem to the control problem for an auxiliary dynamical system-model [2]. Regularization of the problem under consideration is locally realized during the process of choice of positional control in the system-model. The method mentioned above was applied to a number of problems described by some classes of ordinary differential equations [4, 6] as well as by equations with distributed parameters [5]. Different system's characteristics varying in time were under reconstruction, namely, unknown discontinuous inputs, initial and boundary data, distributed disturbances, coefficients of an elliptic operator and so on. In the present paper, using the methods of dynamical identification worked out earlier (see the cited literature), we indicate two algorithms for the reconstruction of nonsmooth inputs acting upon a nonlinear system of the second order. These algorithms are stable with respect to informational noises and calculational errors.

2. STATEMENT OF THE PROBLEMS. APPROACH TO THE SOLUTION

Consider a system described by the equations:

$$\begin{aligned} \dot{x}_1(t) &= k(t)x_2(t) + x_1(t)(\lambda x_2(t) - \nu), \\ \dot{x}_2(t) &= -k(t)x_2(t) - (\lambda x_1(t) + \mu)x_2(t) + \gamma(t), \\ t \in T &= [t_0, \vartheta], \quad x_1(t_0) = x_{10}, \quad x_2(t_0) = x_{20}. \end{aligned} \quad (2.1)$$

This model describes the process of diffusion of innovation [1]. It is assumed that constants λ , ν , μ are known but the function $\gamma(t)$ and (or) the function $k(t)$ are uncertain. We consider the situation when a function $\gamma(t)$ (a measurable Lebesgue function satisfying the condition $\gamma(t) \in P = [-f, f]$, $t \in T$) acts upon the system. Here $f = \text{const} \in (0, +\infty)$. This function as well as the solution of system (2.1) corresponding to it are unknown. At discrete, frequent enough, time moments

$$\tau_i \in \Delta = \{\tau_i\}_{i=0}^m, \quad \tau_{i+1} = \tau_i + \delta, \quad \tau_0 = t_0, \quad \tau_m = \vartheta$$

the value of $z(\tau_i)$ is inaccurately measured. Results of measurements (elements $\xi_i^h \in R^n$) satisfy the inequalities

$$|z(\tau_i) - \xi_i^h|_n \leq h, \quad (2.2)$$

where $h \in (0, 1)$ is a level of informational noise, $|x|_1 = |x|$ is a modulus of the number x , $|y|_2$ is Euclidean norm of the vector $y \in R^2$. We consider two cases. In the first case we assume that at moment τ_i the coordinate $x_1(\tau_i)$ is measured, i.e.

$$z(\tau_i) = x_1(\tau_i), \quad (2.3)$$

and in the second one the pair of these coordinates $x_1(\tau_i)$ and $x_2(\tau_i)$ are measured. Then

$$z(\tau_i) = \{x_1(\tau_i), x_2(\tau_i)\}. \quad (2.4)$$

It is required to indicate (a first case) an algorithm allowing us to reconstruct unknown coordinate $x_2(t)$ and unknown input $\gamma(t)$ (Problem 1) or (a second case) the function $k(t)$ and input $\gamma(t)$ (Problem 2). This is problem being investigated in the present paper.

3. SOLVING METHOD

In Figure 1, the scheme of the solving algorithms for the problems of dynamical reconstruction based on the approach mentioned above is shown.

System (2.1) is accompanied "in real time" by a certain artificial computer-modelled closed-loop control system M with a phase trajectory $w^h(t)$ and a control $u^h(t)$. Then an algorithm forming a feedback control

$$u^h(t) = u^h(t, \xi^h(t), w^h(t)) \quad (3.1)$$

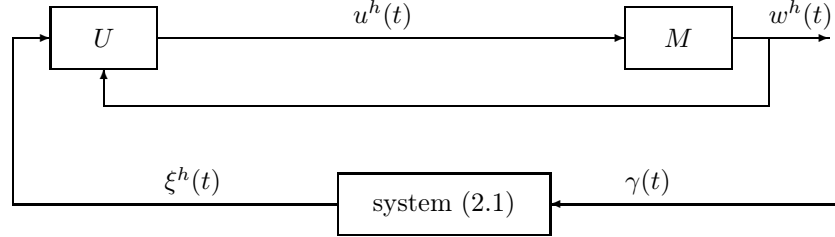


Fig. 1. The scheme of the solving algorithms

for the model M ensuring the output $w^h(t)$ (or the control $u^h(\cdot)$) to estimate in an appropriate sense the unknown parameters $x_2(t)$, $\gamma(t)$ in Problem 1 and $k(t)$, $\gamma(t)$ in Problem 2 is indicated.

Thus, in accordance with the methods described in [3, 4, 6] Problems 1 and 2 may be formulated as follows. In the sequel, a family of partitions

$$\Delta_h = \{\tau_{i,h}\}_{h=0}^{m_h}, \quad \tau_{i+1,h} = \tau_{i,h} + \delta(h), \quad \tau_{0,h} = t_0, \quad \tau_{m_h,h} = \vartheta$$

of the interval T is assumed to be fixed.

Problem 1. It is required to indicate differential equations of the model M

$$\dot{w}^h(t) = f_1(\xi_i^h, w^h(\tau_i), u_i^h), \tag{3.2}$$

$$t \in \delta_{h,i} = [\tau_{i,h}, \tau_{i+1,h}), \quad \tau_i = \tau_{i,h}, \\ w^h(t_0) = w_0^h, \quad w^h(t) \in R^2, \quad w^h = \{w_1^h, w_2^h\},$$

and the rule of choice of controls u_i^h at moments τ_i being a mapping of the form

$$U_1 : \{\tau_i, \xi_i^h, w^h(\tau_i)\} \rightarrow u_i^h = \{u_{i1}^h, u_{i2}^h\} \in R^2 \tag{3.3}$$

such that

$$\int_{t_0}^{\vartheta} |u_1^h(t) - x_2(t)|^2 dt \rightarrow 0, \quad \int_{t_0}^{\vartheta} |u_2^h(t) - \gamma(t)|^2 dt \rightarrow 0 \tag{3.4}$$

as h tends to 0. Here $u^h(t) = \{u_1^h(t), u_2^h(t)\}$, $u_1^h(t) = u_{i1}^h$, $u_2^h(t) = u_{i2}^h$ for $t \in \delta_{h,i}$.

Problem 2. It is required to indicate differential equations of the model M

$$\dot{w}^{h(1)}(t) = f_2(t, \xi_i^h, w^{h(1)}(\tau_i), v_i^h), \tag{3.5}$$

$$t \in \delta_{h,i} = [\tau_{i,h}, \tau_{i+1,h}), \quad w^{h(1)}(t_0) = w_0^{h(1)}, \quad w^{h(1)}(t) \in R^2, \quad w^{h(1)} = \{w_1^{h(1)}, w_2^{h(1)}\},$$

and the rule of choice of control v_i^h at moments τ_i being a mapping of the form

$$U_2 : \{\tau_i, \xi_i^h, w^{h(1)}(\tau_i)\} \rightarrow v_i^h = \{v_{i1}^h, v_{i2}^h\} \in R^2 \tag{3.6}$$

such that

$$\int_{t_0}^{\vartheta} |v_1^h(t) - k(t)|^2 dt \rightarrow 0, \quad \int_{t_0}^{\vartheta} |v_2^h(t) - \gamma(t)|^2 dt \rightarrow 0 \tag{3.7}$$

as h tends to 0. Here $v^h(t) = \{v_{1i}^h, v_{2i}^h\} \quad t \in \delta_{h,i}$.

Following the terminology of [2], the mappings U_1 and U_2 are called *the strategies* (the rules of choice of the system's control (3.2), (3.5)).

4. ALGORITHM FOR SOLVING PROBLEM 1

Let us turn to the description of the algorithm for solving Problem 1. From the above, it is necessary to indicate the model (3.2) and the strategy U_1 (3.3) providing (3.4). Let

$$P(\cdot) = \{u(\cdot) \in L_2(T; R) : u(t) \in P \text{ for a. a. } t \in T\}.$$

From now on, it is assumed that we know a number $K \in (0, +\infty)$ such that each solution $\{x_1(t, u), x_2(t, u)\} (u \in P(\cdot))$ of equation (2.1) satisfies the following conditions

$$\max_{t_0 \leq t \leq \vartheta} |x_1(t, u)| \leq K, \quad \sup_{t_0 \leq t \leq \vartheta} |x_2(t, u)| \leq K. \tag{4.1}$$

Fix some function $\alpha(h) : (0, 1) \rightarrow R^+ = \{r \in R : r \geq 0\}$ with the properties:

$$\alpha(h) \rightarrow 0, \quad \delta(h) \leq h, \quad h^{1/6}/\alpha(h) \rightarrow 0 \text{ as } h \rightarrow 0.$$

This function plays the role of a regularizator (a smoothing functional). Let in (3.2), (3.3)

$$f_1(\xi_i^h, w^h(\tau_i), u_i^h) = \{(k(\tau_i) + \lambda \xi_i^h)u_{i1}^h - \nu \xi_i^h, -(k(\tau_i) + \lambda \xi_i^h + \mu)u_{i1}^h + u_{i2}^h\}, \tag{4.2}$$

$$w_0^h = \{x_{10}, x_{20}\},$$

$$u_{i1}^h = \begin{cases} -\beta_i h^{-2/3} & \text{if } |\beta_i| \leq K h^{2/3}, \\ -K \text{sign} \beta_i, & \text{otherwise,} \end{cases} \tag{4.3}$$

$$u_{i2}^h = \begin{cases} -\beta_i^{(1)} \alpha^{-1}(h), & \text{if } |\beta_i^{(1)}| \leq \alpha(h) f, \\ -f \text{sign} \beta_i^{(1)}, & \text{otherwise.} \end{cases} \tag{4.4}$$

Here

$$\tau_i = \tau_{i,h}, \quad \beta_i = (w_1^h(\tau_i) - \xi_i^h)(k(\tau_i) + \lambda \xi_i^h), \quad \beta_i^{(1)} = w_2^h(\tau_i) - u_{i1}^h, \quad \xi_i^h \in R.$$

We introduce the following condition.

Condition 4.1. (a) Real input $\gamma = \gamma(t)$ generates the solution $x(t) = x(t, \gamma)$ of equation (2.1) such that

$$\inf_{t \in T} |k(t) + \lambda x_1(t, \gamma)| \geq c > 0.$$

- (b) The function $k(t)$ is differentiable and its derivative is an element of the space $L_\infty(T; R)$.
- (c) The function $\gamma(t)/(k(t) + \lambda x_1(t, \gamma))$ has a bounded variation on T .

Theorem 4.2. *Let the condition 4.1 be fulfilled. Then (3.4) take place under the choice of the model equation in the form (3.2), (4.2) and of the strategy U_1 in the form (3.3), (4.3), (4.4).*

Proof. The following inequality follows from results of [3] and conditions 4.1 (a) and 4.1 (b):

$$\int_{t_0}^{\vartheta} |u_1^h(t) - x_2(t)|^2 dt \leq Ch^{1/3}. \tag{4.5}$$

Consider the value

$$\varepsilon(t) = |w_2^h(t) - x_2(t)|^2 + \alpha(h) \int_{t_0}^t \{|u_2^h(\tau)|^2 - |\gamma(\tau)|^2\} d\tau.$$

It is easily seen that for $t \in \delta_i = [\tau_i, \tau_{i+1})$ the inequality is true

$$\begin{aligned} \varepsilon(t) &\leq \varepsilon(\tau_i) + \delta(h) \int_{\tau_i}^t |\dot{w}_2^h(\tau) - \dot{x}_2(\tau)|^2 d\tau + \\ &+ \int_{\tau_i}^t \mu_i(\tau) d\tau + \alpha(h) \int_{\tau_i}^t \{|u_{i2}^h|^2 - |\gamma(\tau)|^2\} d\tau, \end{aligned} \tag{4.6}$$

where

$$\mu_i(t) = 2(w_2^h(\tau_i) - x_2(\tau_i))(\dot{w}_2^h(t) - \dot{x}_2(t)), \quad t \in \delta_i.$$

Consider the value $\mu_i(t)$. We have for $t \in \delta_i$

$$\begin{aligned} \mu_i(t) &= 2(w_2^h(\tau_i) - x_2(\tau_i))\{k(t)x_2(t) - k(\tau_i)u_{i1}^h + \lambda(x_1(t)x_2(t) - \xi_i^h u_{i1}^h) + \\ &+ \mu\lambda(x_2(t) - u_{i1}^h) + u_{i2}^h - \gamma(t)\} \leq \\ &\leq C_1|x_2(\tau_i) - u_{i1}^h| + \sum_{j=1}^5 \lambda_{ji}(t), \quad t \in \delta_i. \end{aligned} \tag{4.7}$$

Here

$$\begin{aligned} \lambda_{1i}(t) &= 2\beta_i^{(1)}(k(t)x_2(t) - k(\tau_i)u_{i1}^h), \\ \lambda_{2i}(t) &= 2\lambda\beta_i^{(1)}(x_1(t)x_2(t) - \xi_i^h u_{i1}^h), \\ \lambda_{3i}(t) &= 2\mu\beta_i^{(1)}(x_2(t) - u_{i1}^h), \\ \lambda_{4i}(t) &= 2\beta_i^{(1)}(u_{i2}^h - \gamma(t)). \end{aligned}$$

Estimate each term in the right-hand part of inequality (4.7). From condition 4.1 (b) it follows that

$$\lambda_{1i}(t) \leq C_2\{|x_2(t) - u_{i1}^h| + \delta(h)\}, \quad t \in \delta_i.$$

Consequently (see (4.5) and (4.1)),

$$\sum_{i=0}^{m_h-1} \int_{\tau_i}^{\tau_{i+1}} \lambda_{1i}(t) dt \leq C_2 \int_{t_0}^{\vartheta} \{|x_2(t) - u_1^h(t)| + \delta(h)\} dt \leq C_3(h^{1/6} + \delta(h)), \quad (4.8)$$

$$\sum_{i=0}^{m_h-1} \delta |x_2(\tau_i) - u_{i1}^h| \leq \int_{t_0}^{\vartheta} |x_2(\tau) - u_1^h(\tau)| d\tau \leq C_5(h^{1/6} + \delta(h)). \quad (4.9)$$

Then, by (2.2), (4.1) we obtain

$$|\xi_i^h - x_1(t)| \leq C_6(h + \delta(h)), \quad t \in \delta_i.$$

Thus,

$$\sum_{i=0}^{m_h-1} \int_{\tau_i}^{\tau_{i+1}} \lambda_{2i}(t) dt \leq C_7(h^{1/6} + \delta(h)). \quad (4.10)$$

By analogy we derive

$$\sum_{i=0}^{m_h-1} \int_{\tau_i}^{\tau_{i+1}} \lambda_{3i}(t) dt \leq C_8(h^{1/6} + \delta(h)). \quad (4.11)$$

Note that

$$\arg \min\{2\beta_i^{(1)}u + \alpha u^2 : u \in P\} = \begin{cases} -\beta_i^{(1)}\alpha^{-1}(h), & \text{if } |\beta_i^{(1)}| \leq \alpha(h)f, \\ -f \operatorname{sign} \beta_i^{(1)}, & \text{otherwise.} \end{cases}$$

Therefore, in virtue of (4.4) we have

$$\begin{aligned} & \int_{\tau_i}^{\tau_{i+1}} \left\{ \lambda_{4i}(\tau) + \alpha(h)\{|u_{i2}^h|^2 - |\gamma(\tau)|^2\} \right\} d\tau = \\ & = \int_{\tau_i}^{\tau_{i+1}} \left\{ \left[2(\beta_i^{(1)}, u_{i2}^h) + \alpha(h)|u_{i2}^h|^2 \right] - \left[2(\beta_i^{(1)}, \gamma(\tau)) + \alpha(h)|\gamma(\tau)|^2 \right] \right\} d\tau \leq 0. \end{aligned} \quad (4.12)$$

Taking into account (4.6)–(4.12) and the inequality $\delta(h) \leq h$ we have for all $i \in [1 : m_h]$ the following estimate

$$\varepsilon(\tau_i) \leq Ch^{1/6}.$$

Further arguments correspond to the standard scheme (see, for example [3, 5]). The theorem is proved. \square

5. ALGORITHM FOR SOLVING PROBLEM 2

Let us turn to the description of the algorithm for solving Problem 2. From the above, it is necessary to indicate the model (3.5) and the strategy U_2 (3.6) providing the convergence result (3.7). Note that in this case $\xi_i = \{\xi_{1i}, \xi_{2i}\} \in R^2$ and inequality (2.2) have the form

$$(|\xi_{1i} - x_1(\tau_i)|^2 + |\xi_{2i} - x_2(\tau_i)|^2)^{1/2} \leq h.$$

Fix some function $\alpha_1(h) : (0, 1) \rightarrow R^+$ (a regularizator) with the properties:

$$\alpha_1(h) \rightarrow 0, \quad \delta(h) \leq h, \quad h^{1/6}/\alpha_1(h) \rightarrow 0 \quad \text{as } h \rightarrow 0.$$

Assume in (3.5), (3.6)

$$f_2(\xi_i^h, v_i^h) = \{v_{i1}^h \xi_{i2} + \lambda \xi_{i1} \xi_{i2} - \lambda v_{i1}^h \xi_{i1}, -v_{i1}^h \xi_{i2} - \lambda \xi_{i1} \xi_{i2} - \lambda \mu \xi_{i2} + v_{i2}^h\}, \quad (5.1)$$

$$w_0^{h(1)} = \{x_{10}, x_{20}\},$$

$$v_{i1}^h = \begin{cases} -\gamma_i^0 h^{-2/3}, & \text{if } |\gamma_i^0| \leq h^{2/3} A, \\ -A \text{sign} \gamma_i^0, & \text{otherwise,} \end{cases} \quad (5.2)$$

$$v_{i2}^h = \begin{cases} -\gamma_i \alpha_1^{-1}(h), & \text{if } |\gamma_i| \leq \alpha_1(h) f, \\ -f \text{sign} \gamma_i, & \text{otherwise.} \end{cases} \quad (5.3)$$

Here

$$\tau_i = \tau_{i,h}, \quad \gamma_i^0 = \xi_{i2}(w_1^h(\tau_i) - \xi_{i1}^h), \quad \gamma_i = w_2^h(\tau_i) - \xi_{i2}^h.$$

Condition 5.1. (a) Real input $\gamma = \gamma(t)$ generates the solution $x(t) = x(t, \gamma)$ of equation (2.1) such that

$$x_2(t) \geq c > 0, \quad t \in T.$$

(b) The function $k(t)$ satisfies condition 4.1 (b) and the following inequality

$$|k(t)| \leq A, \quad t \in T.$$

Theorem 5.2. *Let condition 5.1 is fulfilled. Then the convergence result (3.7) takes place under the choice of the model equation in the form (3.5), (5.1) and of the strategy U_2 in the form (3.6), (5.3).*

Proof. The proof of this theorem is performed by the scheme used in Theorem 4.2. In the beginning we estimate a variation of the value

$$\varepsilon_1(t) = |w_1^{h(1)}(t) - x_1(t)|^2.$$

We have

$$\varepsilon_1(\tau_{i+1}) \leq \varepsilon_1(\tau_i) + \left| \int_{\tau_i}^{\tau_{i+1}} (\dot{w}_1^{h(1)}(\tau) - \dot{x}_1(\tau)) d\tau \right|^2 +$$

$$+2(w_1^{h(1)}(\tau_i) - x_1(\tau_i)) \int_{\tau_i}^{\tau_{i+1}} \{v_{i1}^h \xi_{i2} + \lambda \xi_{i1} \xi_{i2} - \lambda \nu \xi_{i1} - k(t)x_2(t) - \lambda x_1(t)x_2(t) + \lambda \nu x_1(t)\} dt.$$

In virtue of the boundedness of $k(t)$ and $\gamma(t)$ we have

$$\sup_{t \in T} |\dot{w}_1^h(t)| \leq K_1, \quad \sup_{t \in T} |\dot{x}_1(t)| \leq K_2.$$

Therefore

$$\varepsilon_1(\tau_{i+1}) \leq \varepsilon_1(\tau_i) + d_0(\delta(h) + h)\delta(h) + \sum_{j=1}^3 \int_{\tau_i}^{\tau_{i+1}} \lambda_i^{(j)}(\tau) d\tau,$$

when

$$\begin{aligned} \lambda_i^{(1)}(\tau) &= 2\gamma_i^0(v_{i1}^h - k(\tau)) + 2(w_1^h(\tau_i) - \xi_{i1}^h)(\xi_{i2}^h - x_2(\tau)), \\ \lambda_i^{(2)}(\tau) &= 2\lambda(w_1^h(\tau_i) - \xi_{i1}^h)(\xi_{i1}^h \xi_{i2}^h - x_1(\tau)x_2(\tau)), \\ \lambda_i^{(3)}(\tau) &= 2\lambda\nu(w_1^h(\tau_i) - \xi_{i1}^h)(x_1(\tau) - \xi_{i1}^h). \end{aligned}$$

Analogously in (4.12) by virtue of (5.2) we obtain

$$\int_{\tau_i}^{\tau_{i+1}} \left\{ 2\gamma_i^0(v_{i1}^h - k(\tau)) + h^{2/3} \{|v_{i1}^h|^2 - |k(\tau)|^2\} \right\} d\tau \leq 0.$$

Therefore

$$\int_{\tau_i}^{\tau_{i+1}} \left\{ \lambda_i^{(1)}(\tau) + h^{2/3} \{|v_{i1}^h|^2 - |k(\tau)|^2\} \right\} d\tau \leq d_1(h + \delta(h))\delta(h).$$

It is easily seen that

$$\begin{aligned} \int_{\tau_i}^{\tau_{i+1}} \lambda_i^{(2)}(\tau) d\tau &\leq d_2(h + \delta(h))\delta(h), \\ \int_{\tau_i}^{\tau_{i+1}} \lambda_i^{(3)}(\tau) d\tau &\leq d_3(h + \delta(h))\delta(h). \end{aligned}$$

Consequently,

$$\varepsilon_1(\tau_{i+1}) \leq \varepsilon_1(\tau_i) + d_3\delta(h)(h + \delta(h)).$$

From this we derive

$$\varepsilon_1(\tau_{i+1}) \leq C(h + \delta(h)), \quad i \in [0 : m_h - 1]. \quad (5.4)$$

From this, following [3, 5], we deduce

$$\int_{t_0}^{\vartheta} |v_1^h(t) - k(t)|^2 dt \leq Ch^{1/3}. \tag{5.5}$$

Consider the value

$$\varepsilon_2(t) = |w_2^{h(1)}(t) - x_2(t)|^2 + \alpha_1(h) \int_{t_0}^t \{|v_2^h(\tau)|^2 - |\gamma(\tau)|^2\} d\tau.$$

It is easily seen that for $t \in \delta_i = [\tau_i, \tau_{i+1})$ the inequality

$$\varepsilon_2(t) \leq \varepsilon_2(\tau_i) + \int_{\tau_i}^t \left\{ \delta(h) |\dot{w}_2^{h(1)}(\tau) - \dot{x}_2(\tau)|^2 + \nu_i(\tau) + \alpha_1(h) \left(|v_{i2}^h|^2 - |\gamma(\tau)|^2 \right) \right\} d\tau$$

is true. Here

$$\begin{aligned} \nu_i(t) &= \varphi_i(\dot{w}_2^{h(1)}(t) - \dot{x}(t)), \quad t \in \delta_i, \\ \varphi_i &= 2(w_2^{h(1)}(\tau_i) - \dot{x}(\tau_i)). \end{aligned}$$

Consider the value $\nu_i(t)$. For $t \in \delta_i$ from (2.1), (3.5), (4.2) we get

$$\nu_i(t) = \sum_{j=1}^n \gamma_{ji}(t), \tag{5.6}$$

$$\begin{aligned} \gamma_{1i}(t) &= \varphi_i(k(t)x_2(t) - v_{i1}^h \xi_{i2}), \\ \gamma_{2i}(t) &= \lambda \varphi_i(x_1(t)x_2(t) - \xi_{i1} \xi_{i2}), \\ \gamma_{3i}(t) &= \lambda \mu \varphi_i(x_2(t) - \xi_{i2}), \\ \gamma_{4i}(t) &= 2\varphi_i(v_{i2}^h - \gamma(t)). \end{aligned}$$

Using (5.5), we deduce that

$$\sum_{i=0}^{m_h-1} \gamma_{1i}(t) \leq C_1(h^{1/6} + \delta). \tag{5.7}$$

It is easily seen that

$$\sum_{i=0}^{m_h-1} \int_{\tau_i}^{\tau_{i+1}} (\gamma_{2i}(t) + \gamma_{3i}(t)) dt \leq C_2(h + \delta). \tag{5.8}$$

Note that

$$\arg \min\{2\gamma_i u + \alpha_1 u^2 : |u| \leq f\} = \begin{cases} -\gamma_i \alpha_1^{-1}(h), & \text{if } |\gamma_i| \leq \alpha_1(h)f, \\ -f \operatorname{sign} \gamma_i, & \text{otherwise.} \end{cases}$$

By analogy with (4.12) we derive

$$\int_{\tau_i}^{\tau_{i+1}} \{\gamma_{4i}(t) + \alpha_1(t)\{|v_{i2}^h|^2 - |\gamma(t)|^2\}\} dt \leq C_3(h + \delta)\delta. \quad (5.9)$$

Taking into account (5.6)–(5.9), we get for all $i \in [1 : m_h]$ the estimate

$$\varepsilon_2(\tau_i) \leq C(h^{1/6} + \delta).$$

The assertion of theorem 5.2 follows from this inequality. \square

Example 5.3. The algorithm for solving Problem 1 was tested by a model example. We considered system (2.1) on time interval $T = [0, 2]$, $x_1, x_2 \in R$. It was assumed that the initial state has the form $x_1(t) = 1$, $x_2(t) = 2$. The input was computed by the following formulas

$$\gamma(t) = 1 + t.$$

At moments τ_i the value $\xi_{i1} = x_1(\tau_i) + h \cos(M\tau_i)$, was measured. A model with initial state $w_1(0) = 1 + h$, $w_2(0) = 2 - h$ and controls we took according (3.2), (4.2), (4.3), (4.4). In Figures 2–4 the results of calculations are presented for the case when $k(t) = \operatorname{const} = 0.5$, $\lambda = 3$, $\nu = 0.1$, $\mu = 1$, $f = 3$, $M = 10$. Figure 2 corresponds to the case when $h = 0.001$, Figure 3 – $h = 0.1$, Figure 4 – $h = 0.01$. In Figures 2–4 solid (dashed) lines represent model controls $v_{1,2}(t)$ (the second coordinate of the system (2.1) and the real control).

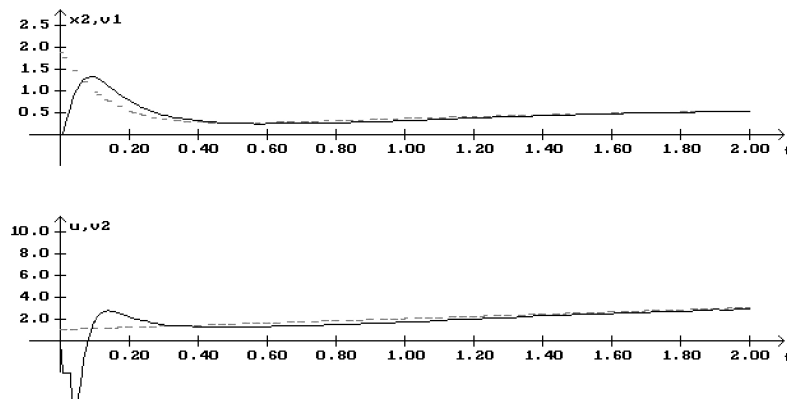


Fig. 2. $h = 0.001$

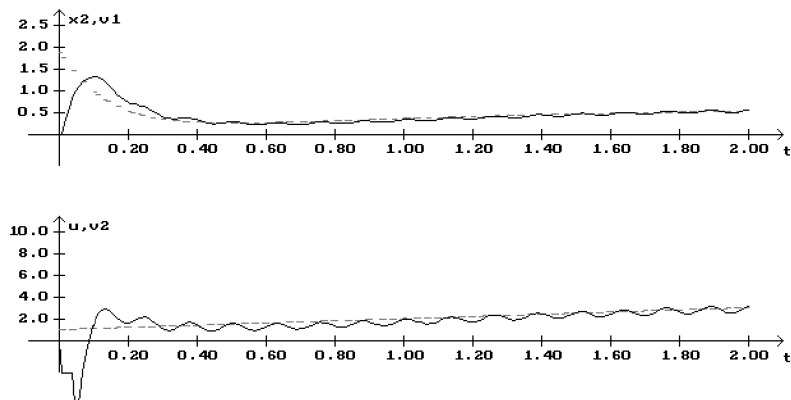


Fig. 3. $h = 0.01$

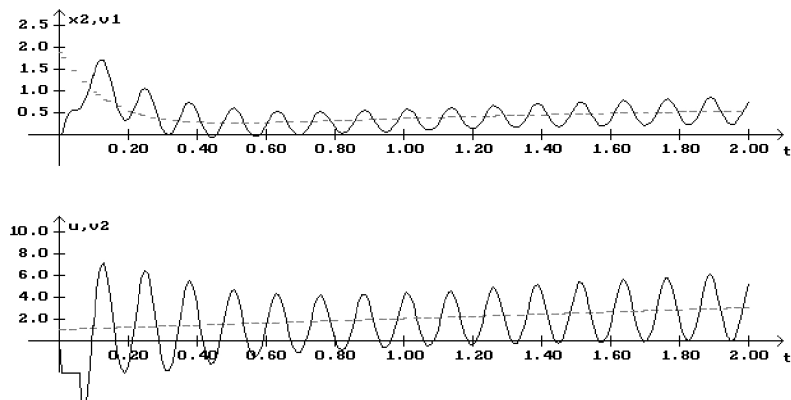


Fig. 4. $h = 0.1$

Acknowledgments

The work was partially supported by the Russian Foundation for Basic Research (Project 09-01-00378), the Program on Basic Research of the Presidium of the Russian Acad. Sci. (Project “Mathematical Control Theory”, 09-P-1-1013), the Ural-Siberian Integration Project (09-C-1-1010), and the Program for support of leading scientific schools of Russia (no 65590.2010.1).

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Received: March 24, 2010.

Accepted: May 4, 2010.