Opuscula Mathematica • Vol. 23 • 2003 • pp. 99-115

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ON SOME COMPLEX SPLINE OPERATORS

Abstract. The paper is concerned with the space $S_n(\Delta_N)$ of splines in the complex (or real) variable z of degree n with respect to a given partition Δ_N of a rectifiable Jordan curve Γ. We define an operator $Q_N : L_p(\Gamma) \to S_n(\Delta_N)$, such that $Q_N f = f$ for $f \in S_n(\Delta_N)$, by means of a system of step functions "biorthogonal" to B-splines and then we estimate the order of approximation of f by $Q_N f$ in the space $C^k(\Gamma)$, $k \leq n$. We apply the obtained results to approximation of analytic functions in the interior D of a Jordan curve Γ and of class C^k on $\bar{D}$ $(k = 0, ..., n-1)$ by analytic splines defined in the interior Γ by means of the Cauchy integral. Then we consider the special case, where Γ is the interval $[0, 1]$ and we estimate the order of approximation of f by $Q_N f$ in the space $W_p^n([0, 1])$ for $1 \leq p \leq \infty$.

Keywords: operator associated with step functions, real and complex B-splines, analytic splines, analytic functions, approximation.

Mathematics Subject Classification: 41A15, 41A35, 41A17.