

MULTIPLICATIVE ZAGREB INDICES AND COINDICES OF SOME DERIVED GRAPHS

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Abstract. In this note, we obtain the expressions for multiplicative Zagreb indices and coindices of derived graphs such as a line graph, subdivision graph, vertex-semitotal graph, edge-semitotal graph, total graph and paraline graph.

Keywords: multiplicative Zagreb indices and coindices, derived graphs.

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1. INTRODUCTION

In this paper, we are concerned with simple graphs without isolated vertices. Let G be such a graph with vertex set $V(G)$, $|V(G)| = n$, and edge set $E(G)$, $|E(G)| = m$. As usual, n is the order and m the size of G . The degree of a vertex $w \in V(G)$ is the number of vertices adjacent to w and is denoted by $d_G(w)$. A vertex $w \in V(G)$ is said to be pendant if $d_G(w) = 1$. The degree of an edge $e = uv$ in G , denoted by $d_G(e)$, is defined by $d_G(e) = d_G(u) + d_G(v) - 2$. We refer to [9] for unexplained terminology and notation.

A graphical invariant is a number related to a graph, in other words, it is a fixed number under graph automorphisms. In chemical graph theory, these invariants are also called the topological indices. In 1984, Narumi and Katayama [11] considered the product index as

$$NK(G) = \prod_{u \in V(G)} d_G(u)$$

for representing the carbon skeleton of a saturated hydrocarbon, and named it as a simple topological index. Tomović and Gutman renamed this molecular structure

descriptor as the Narumi-Katayama index [15]. In 2010, Todeshine *et al.* [13, 14] proposed the multiplicative variants of ordinary Zagreb indices, which are defined as follows:

$$\prod_1(G) = \prod_{u \in V(G)} d_G(u)^2 = [NK(G)]^2 \quad \text{and} \quad \prod_2(G) = \prod_{uv \in E(G)} d_G(u)d_G(v).$$

These two graph invariants are called first and second multiplicative Zagreb indices by Gutman [6]. And recently, Eliasi *et al.* [5] introduced a further multiplicative version of the first Zagreb index as

$$\prod_1^*(G) = \prod_{uv \in E(G)} [d_G(u) + d_G(v)].$$

In [18] and [7] the authors called it a multiplicative sum Zagreb index and modified first multiplicative Zagreb index respectively. The second multiplicative Zagreb index for any graph G can also be written as [6]

$$\prod_2(G) = \prod_{u \in V(G)} d_G(u)^{d_G(u)}.$$

Xu *et al.* [19] defined the first and second multiplicative Zagreb coindices, respectively, as

$$\overline{\prod}_1(G) = \prod_{uv \notin E(G)} [d_G(u) + d_G(v)] \quad \text{and} \quad \overline{\prod}_2(G) = \prod_{uv \notin E(G)} d_G(u)d_G(v).$$

The main properties of multiplicative Zagreb indices are summarized in [1, 4, 10, 12, 17, 18, 20].

We introduce the modified second multiplicative Zagreb index as

$$\prod_2^*(G) = \prod_{uv \in E(G)} [d_G(u) + d_G(v)]^{[d_G(u) + d_G(v)]}.$$

2. DERIVED GRAPHS

In recent papers [2, 3, 8], the authors obtained the expressions for Zagreb indices and coindices of derived graphs. This motivates us to find expressions for \prod_1 , \prod_2 , \prod_1^* and $\overline{\prod}_2$ of derived graphs.

Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. We are concerned with the following graphs derived from G ([8]):

- *line graph* $L = L(G)$; $V(L) = E(G)$ and the two vertices of L are adjacent if the corresponding edges of G are incident with a common vertex;
- *subdivision graph* $S = S(G)$; $V(S) = V(G) \cup E(G)$ and the vertex of S corresponding to the edge uv of G is inserted in the edge uv of G ;

- vertex-semi-total graph $T_2 = T_2(G)$; $V(T_2) = V(G) \cup E(G)$ and $E(T_2) = E(S) \cup E(G)$;
- edge-semi-total graph $T_1 = T_1(G)$; $V(T_1) = V(G) \cup E(G)$ and $E(T_1) = E(S) \cup E(L)$;
- total graph $T = T(G)$; $V(T) = V(G) \cup E(G)$ and $E(T) = E(S) \cup E(G) \cup E(L)$;
- paraline graph $PL = PL(G)$ is the line graph of the subdivision graph.

In Figure 1 self-explanatory examples of these derived graphs are depicted.

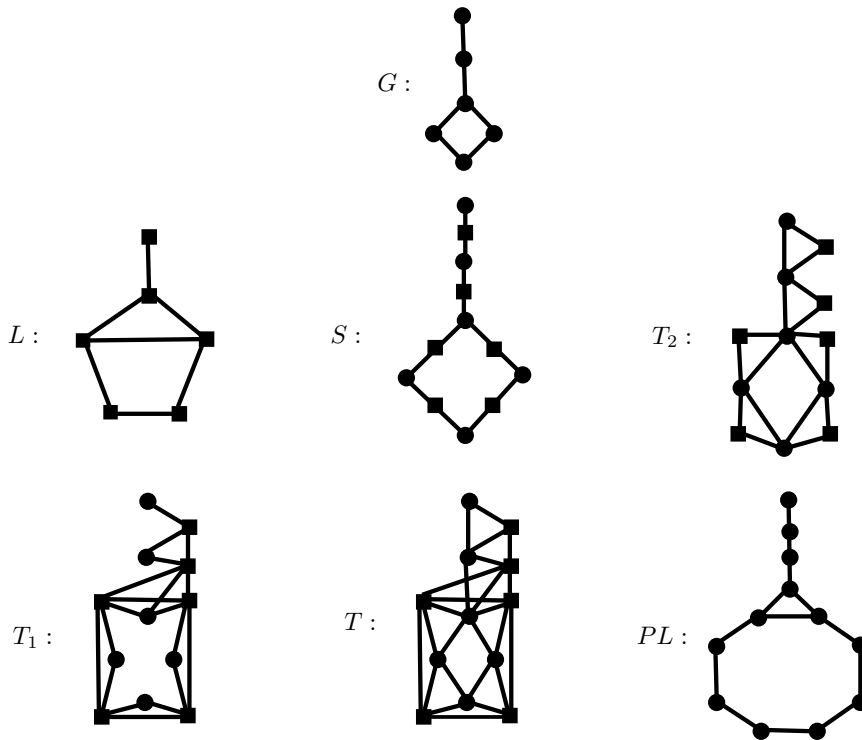


Fig. 1. Various graphs derived from the graph G . The vertices of these derived graphs (except the paraline graph PL), corresponding to the vertices of the parent graph G , are indicated by circles. The vertices of these graphs corresponding to the edges of the parent graph G are indicated by squares

In [19], Kexiang Xu *et al.* obtained the expressions for $\overline{\prod}_2(G)$ of any connected graph G as

$$\overline{\prod}_2(G) = \prod_{u \in V(G)} d_G(u)^{n-1-d_G(u)} \quad \text{and} \quad \prod_2(G) \overline{\prod}_2(G) = \left(\prod_1(G) \right)^{\frac{n-1}{2}}$$

which are not satisfied for a complete graph. The following lemmas give the correct expressions for $\overline{\prod}_2(G)$.

Lemma 2.1. For a connected graph $G \neq K_n$, we have

$$\overline{\prod}_2(G) = \prod_{u \in V(G)} d_G(u)^{n-1-d_G(u)}.$$

Lemma 2.2. For a connected graph $G \neq K_n$, we have

$$\prod_2(G) \overline{\prod}_2(G) = \left(\prod_1(G) \right)^{\frac{n-1}{2}}.$$

Next we present the values of multiplicative Zagreb indices and coindices for several classes of graphs.

Example 2.3. Let P_n be the path with n vertices. The pendant vertices have degree 1 and other vertices have degree 2. Hence,

- (i) $\prod_1(P_n) = 4^{(n-2)}$,
- (ii) $\overline{\prod}_2(P_n) = 4^{(n-2)}$,
- (iii) $\prod_1^*(P_n) = 9 \cdot 4^{(n-3)}$, $n \geq 3$,
- (iv) $\overline{\prod}_1(P_n) = 2 \cdot 9^{(n-3)} \cdot 4^{(n-4)!}$, $n \geq 4$,
- (v) $\overline{\prod}_2(P_n) = 2^{(n-2)(n-3)}$, $n \geq 3$,
- (vi) $\prod_2^*(P_n) = 3^6 \cdot 4^{4(n-3)}$, $n \geq 3$.

Example 2.4. Consider the cycle C_n with n vertices. Since its every vertex is of degree 2, then

- (i) $\prod_1(C_n) = 4^n$,
- (ii) $\overline{\prod}_2(C_n) = 4^n$,
- (iii) $\prod_1^*(C_n) = 4^n$,
- (iv) $\overline{\prod}_1(C_n) = 4^{\frac{n(n-3)}{2}}$, $n \geq 4$,
- (v) $\overline{\prod}_2(C_n) = 4^{\frac{n(n-3)}{2}}$, $n \geq 4$,
- (vi) $\prod_2^*(C_n) = 4^{4n}$.

Example 2.5. Let K_n be the complete graph on n vertices. All vertices of K_n have degree $n - 1$ and so

- (i) $\prod_1(K_n) = (n - 1)^{2n}$, $n \geq 2$,
- (ii) $\overline{\prod}_2(K_n) = (n - 1)^{n(n-1)}$, $n \geq 2$,
- (iii) $\prod_1^*(K_n) = [2(n - 1)]^{\frac{n(n-1)}{2}}$, $n \geq 2$,
- (iv) $\overline{\prod}_1(K_n) = 0$,
- (v) $\overline{\prod}_2(K_n) = 0$,
- (vi) $\prod_2^*(K_n) = [2(n - 1)]^{n(n-1)^2}$, $n \geq 2$.

Example 2.6. Let $K_{r,s}$ be the complete bipartite graph. Then $K_{r,s}$ has $r + s$ vertices and rs edges. Hence,

- (i) $\prod_1(K_{r,s}) = r^{2s} \cdot s^{2r}$,
- (ii) $\prod_2(K_{r,s}) = [rs]^{rs}$,
- (iii) $\prod_1^*(K_{r,s}) = [r + s]^{rs}$,
- (iv) $\overline{\prod}_1(K_{r,s}) = [2r]^{\frac{s(s-1)}{2}} \cdot [2s]^{\frac{r(r-1)}{2}}$, $r \neq 1$ and $s \neq 1$,
- (v) $\overline{\prod}_2(K_{r,s}) = r^{s(s-1)} \cdot s^{r(r-1)}$, $r \neq 1$ and $s \neq 1$,
- (vi) $\overline{\prod}_2^*(K_{r,s}) = [r + s]^{rs(r+s)}$.

Example 2.7. Let W_n be the wheel on n vertices. Its central vertex has degree $n - 1$ and its other vertices have degree 3. This implies

- (i) $\prod_1(W_n) = (n - 1)^2 \cdot 3^{2(n-1)}$,
- (ii) $\prod_2(W_n) = [3(n - 1)]^{(n-1)} \cdot 3^{2(n-1)}$,
- (iii) $\prod_1^*(W_n) = [n + 2]^{(n-1)} \cdot 6^{(n-1)}$,
- (iv) $\overline{\prod}_1(W_n) = 6^{\frac{(n-1)(n-4)}{2}}$, $n \geq 5$,
- (v) $\overline{\prod}_2(W_n) = 9^{\frac{(n-1)(n-4)}{2}}$, $n \geq 5$,
- (vi) $\overline{\prod}_2^*(W_n) = 6^{6(n-1)} \cdot [n + 2]^{(n-1)(n+2)}$.

3. RESULTS

Theorem 3.1. *Let G be a graph of order n and size m . Then*

$$\prod_1(S) = 4^m \prod_1(G).$$

Proof. Note that S has $n + m$ vertices.

$$\prod_1(S) = \prod_{u \in V(S)} d_S(u)^2 = \prod_{u \in V(S) \cap V(G)} d_S(u)^2 \prod_{e \in V(S) \cap E(G)} d_S(e)^2.$$

Note that for $u \in V(S) \cap V(G)$, $d_S(u) = d_G(u)$ and for $e \in V(S) \cap E(G)$, $d_S(e) = 2$.

$$\prod_1(S) = \prod_{u \in V(G)} d_G(u)^2 \prod_{e \in E(G)} 2^2 = 4^m \prod_1(G). \quad \square$$

Theorem 3.2. *Let G be a graph of order n and size m . Then*

$$\prod_2(S) = 4^m \prod_2(G).$$

Proof. Since S has $n + m$ vertices, then

$$\prod_2(S) = \prod_{u \in V(S)} d_S(u)^{d_S(u)} = \prod_{u \in V(S) \cap V(G)} d_S(u)^{d_S(u)} \prod_{e \in V(S) \cap E(G)} d_S(e)^{d_S(e)}.$$

Since for $u \in V(S) \cap V(G)$, $d_S(u) = d_G(u)$ and for $e \in V(S) \cap E(G)$, $d_S(e) = 2$.

$$\prod_2(S) = \prod_{u \in V(G)} d_G(u)^{d_G(u)} \prod_{e \in E(G)} 2^2 = 4^m \prod_2(G). \quad \square$$

Theorem 3.3. *Let G be a graph of order n and size m . Then*

$$\prod_1^*(S) = \prod_{u \in V(G)} [2 + d_G(u)]^{d_G(u)}.$$

Proof. Since S has $n + m$ vertices, then we have

$$\prod_1^*(S) = \prod_{uv \in E(S)} [d_S(u) + d_S(v)].$$

Since for $u \in V(S) \cap V(G)$, $d_S(u) = d_G(u)$ and for $e \in V(S) \cap E(G)$, $d_S(e) = 2$.

$$\prod_1^*(S) = \prod_{u \in V(G)} [2 + d_G(u)]^{d_G(u)}. \quad \square$$

Corollary 3.4. *Let G be a connected graph of order n and size m . Then*

$$\overline{\prod}_2(S) = \frac{2^{m^2 + nm - 3m} [\prod_1(G)]^{\frac{n+m-1}{2}}}{\prod_2(G)}.$$

Proof. From Lemma 2.2 we have

$$\overline{\prod}_2(S) = \frac{[\prod_1(S)]^{\frac{n+m-1}{2}}}{\prod_2(S)}.$$

From Theorems 3.1 and 3.2 we get the result. □

Theorem 3.5. *Let G be a graph of order n and size m . Then*

$$\prod_1(T_2) = 4^{n+m} \prod_1(G).$$

Proof. Note that T_2 has $n + m$ vertices.

$$\prod_1(T_2) = \prod_{u \in V(T_2)} d_{T_2}(u)^2 = \prod_{u \in V(T_2) \cap V(G)} d_{T_2}(u)^2 \prod_{e \in V(T_2) \cap E(G)} d_{T_2}(e)^2.$$

Note that for $u \in V(T_2) \cap V(G)$, $d_{T_2}(u) = 2d_G(u)$ and for $e \in V(T_2) \cap E(G)$, $d_{T_2}(e) = 2$.

$$\prod_1(T_2) = \prod_{u \in V(G)} [2d_G(u)]^2 \prod_{e \in E(G)} 2^2 = 4^{n+m} \prod_1(G). \quad \square$$

Theorem 3.6. *Let G be a graph of order n and size m . Then*

$$\prod_2(T_2) = 64^m \prod_1(G) \prod_2(G).$$

Proof. Since T_2 has $n + m$ vertices and $3m$ edges, then we have

$$\begin{aligned} \prod_2(T_2) &= \prod_{uv \in E(T_2)} d_{T_2}(u)d_{T_2}(v) \\ &= \prod_{uv \in E(T_2) \cap E(G)} d_{T_2}(u)d_{T_2}(v) \prod_{ue \in E(T_2) \setminus E(G)} d_{T_2}(u)d_{T_2}(e). \end{aligned}$$

Since for $u \in V(T_2) \cap V(G)$, $d_{T_2}(u) = 2d_G(u)$ and for $e \in V(T_2) \cap E(G)$, $d_{T_2}(e) = 2$.

$$\begin{aligned} \prod_2(T_2) &= \prod_{uv \in E(G)} 2d_G(u)2d_G(v) \prod_{ue \in E(T_2) \setminus E(G)} (2)2d_G(u) \\ &= 4^m \prod_{uv \in E(G)} d_G(u)d_G(v) 4^{2m} \prod_{u \in V(G)} d_G(u)^2 \\ &= 64^m \prod_1(G) \prod_2(G). \quad \square \end{aligned}$$

Theorem 3.7. *Let G be a graph of order n and size m . Then*

$$\prod_1^*(T_2) = 8^m \prod_1^*(G) \prod_{u \in V(G)} [1 + d_G(u)]^{d_G(u)}.$$

Proof. Since T_2 has $n + m$ vertices and $3m$ edges, then we have

$$\begin{aligned} \prod_1^*(T_2) &= \prod_{uv \in E(T_2)} [d_{T_2}(u) + d_{T_2}(v)] \\ &= \prod_{uv \in E(T_2) \cap E(G)} [d_{T_2}(u) + d_{T_2}(v)] \prod_{ue \in E(T_2) \setminus E(G)} [d_{T_2}(u) + d_{T_2}(e)]. \end{aligned}$$

Since for $u \in V(T_2) \cap V(G)$, $d_{T_2}(u) = 2d_G(u)$ and for $e \in V(T_2) \cap E(G)$, $d_{T_2}(e) = 2$.

$$\begin{aligned} \prod_1^*(T_2) &= \prod_{uv \in E(G)} [2d_G(u) + 2d_G(v)] \prod_{ue \in E(T_2) \setminus E(G)} [2 + 2d_G(u)] \\ &= 2^m \prod_{uv \in E(G)} [d_G(u) + d_G(v)] 2^{2m} \prod_{u \in V(G)} [1 + d_G(u)]^{d_G(u)} \\ &= 8^m \prod_1^*(G) \prod_{u \in V(G)} [1 + d_G(u)]^{d_G(u)}. \quad \square \end{aligned}$$

Corollary 3.8. *Let G be a connected graph of order n and size m . Then*

$$\overline{\prod}_2(T_2) = \frac{2^{(n+m)^2 - n - 7m} [\prod_1(G)]^{\frac{n+m-3}{2}}}{\prod_2(G)}.$$

Proof. From Lemma 2.2 we have

$$\overline{\prod}_2(T_2) = \frac{[\prod_1(T_2)]^{\frac{n+m-1}{2}}}{\prod_2(T_2)}.$$

From Theorems 3.5 and 3.6 we get the result. \square

Theorem 3.9. *Let G be a graph of order n and size m . Then*

$$\prod_1(T_1) = \prod_1(G) \left[\prod_1^*(G) \right]^2.$$

Proof. Note that T_1 has $n + m$ vertices.

$$\begin{aligned} \prod_1(T_1) &= \prod_{u \in V(T_1)} d_{T_1}(u)^2 \\ &= \prod_{u \in V(T_1) \cap V(G)} d_{T_1}(u)^2 \prod_{e_i \in V(T_1) \cap E(G)} d_{T_1}(e_i)^2. \end{aligned}$$

Note that for $u \in V(T_1) \cap V(G)$, $d_{T_1}(u) = d_G(u)$ and for $e_i \in V(T_1) \cap E(G)$, $d_{T_1}(e_i) = d_G(u_i) + d_G(v_i)$.

$$\begin{aligned} \prod_1(T_1) &= \prod_{u \in V(G)} d_G(u)^2 \prod_{u_i v_i \in E(G)} [d_G(u_i) + d_G(v_i)]^2 \\ &= \prod_1(G) \left[\prod_1^*(G) \right]^2. \end{aligned} \quad \square$$

Theorem 3.10. *Let G be a graph of order n and size m . Then*

$$\prod_2(T_1) = \prod_2(G) \prod_2^*(G).$$

Proof. Since T_1 has $n + m$ vertices, then we have

$$\begin{aligned} \prod_2(T_1) &= \prod_{u \in V(T_1)} d_{T_1}(u)^{d_{T_1}(u)} \\ &= \prod_{u \in V(T_1) \cap V(G)} d_{T_1}(u)^{d_{T_1}(u)} \prod_{e_i \in V(T_1) \cap E(G)} [d_{T_1}(e_i)]^{[d_{T_1}(e_i)]}. \end{aligned}$$

Since for $u \in V(T_1) \cap V(G)$, $d_{T_1}(u) = d_G(u)$ and for $e_i \in V(T_1) \cap E(G)$, $d_{T_1}(e_i) = d_G(u_i) + d_G(v_i)$.

$$\begin{aligned} \prod_2(T_1) &= \prod_{u \in V(G)} d_G(u)^{d_G(u)} \prod_{u_i v_i \in E(G)} [d_G(u_i) + d_G(v_i)]^{[d_G(u_i) + d_G(v_i)]} \\ &= \prod_2(G) \prod_2^*(G). \end{aligned} \quad \square$$

Corollary 3.11. *Let G be a connected graph of order n and size m . Then*

$$\overline{\prod}_2(T_1) = \frac{[\prod_1(G)]^{\frac{n+m-1}{2}} [\prod_1^*(G)]^{n+m-1}}{\prod_2(G) \prod_2^*(G)}.$$

Proof. From Lemma 2.2 we have

$$\overline{\prod}_2(T_1) = \frac{[\prod_1(T_1)]^{\frac{n+m-1}{2}}}{\prod_2(T_1)}.$$

From Theorems 3.9 and 3.10 we get the result. □

In [16], an incorrect expression for $\prod_1(T)$ was established. The following theorem gives the correct expression for $\prod_1(T)$.

Theorem 3.12. *Let G be a graph of order n and size m . Then*

$$\prod_1(T) = 4^n \prod_1(G) \left[\prod_1^*(G) \right]^2.$$

Proof. Note that T has $n + m$ vertices.

$$\prod_1(T) = \prod_{u \in V(T)} d_T(u)^2 = \prod_{u \in V(T) \cap V(G)} d_T(u)^2 \prod_{e_i \in V(T) \cap E(G)} d_T(e_i)^2.$$

Note that for $u \in V(T) \cap V(G)$, $d_T(u) = 2d_G(u)$ and for $e_i \in V(T) \cap E(G)$, $d_T(e_i) = d_G(u_i) + d_G(v_i)$.

$$\prod_1(T) = \prod_{u \in V(G)} [2d_G(u)]^2 \prod_{u_i v_i \in E(G)} [d_G(u_i) + d_G(v_i)]^2 = 4^n \prod_1(G) \left[\prod_1^*(G) \right]^2. \quad \square$$

Theorem 3.13. *Let G be a graph of order n and size m . Then*

$$\prod_2(T) = 16^m \prod_2^*(G) \left[\prod_2(G) \right]^2.$$

Proof. Since T has $n + m$ vertices, then we have

$$\prod_2(T) = \prod_{u \in V(T)} d_T(u)^{d_T(u)} = \prod_{u \in V(T) \cap V(G)} d_T(u)^{d_T(u)} \prod_{e_i \in V(T) \cap E(G)} d_T(e_i)^{d_T(e_i)}.$$

Note that for $u \in V(T) \cap V(G)$, $d_T(u) = 2d_G(u)$ and for $e_i \in V(T) \cap E(G)$, $d_T(e_i) = d_G(u_i) + d_G(v_i)$.

$$\begin{aligned} \prod_2(T) &= \prod_{u \in V(G)} [2d_G(u)]^{[2d_G(u)]} \prod_{u_i v_i \in E(G)} [d_G(u_i) + d_G(v_i)]^{[d_G(u_i) + d_G(v_i)]} \\ &= \prod_{u \in V(G)} 2^{[2d_G(u)]} [d_G(u)]^{2d_G(u)} \prod_2^*(G) \\ &= 16^m \prod_2^*(G) \left[\prod_2(G) \right]^2. \quad \square \end{aligned}$$

Corollary 3.14. *Let G be a connected graph of order n and size m . Then*

$$\overline{\prod}_2(T) = \frac{2^{n(n+m-1)-4m} [\prod_1(G)]^{\frac{n+m-1}{2}} [\prod_1^*(G)]^{n+m-1}}{\prod_2^*(G) [\prod_2(G)]^2}.$$

Proof. From Lemma 2.2 we have

$$\overline{\prod}_2(T) = \frac{[\prod_1(T)]^{\frac{n+m-1}{2}}}{\prod_2(T)}.$$

From Theorems 3.12 and 3.13 we get the result. □

Theorem 3.15. *Let G be a graph of order n and size m . Then*

$$\prod_1(PL) = \left[\prod_2(G) \right]^2.$$

Proof. Note that paraline graph PL has $2m$ vertices, and $d_G(u)$ of its vertices have the same degree as the vertex u of the graph G .

$$\prod_1(PL) = \prod_{u \in V(PL)} d_{PL}(u)^2 = \prod_{u \in V(G)} d_G(u)^{2d_G(u)} = \left[\prod_2(G) \right]^2. \quad \square$$

Theorem 3.16. *Let G be a graph of order n and size m . Then*

$$\prod_2(PL) = \prod_{u \in V(G)} [d_G(u)]^{[d_G(u)]^2}.$$

Proof. Since PL has $2m$ vertices, then we have

$$\begin{aligned} \prod_2(PL) &= \prod_{uv \in E(PL)} d_{PL}(u)d_{PL}(v) \\ &= \prod_2(G) \prod [d_G(u)]^{[d_G(u)(d_G(u)-1)]} = \prod_{u \in V(G)} [d_G(u)]^{[d_G(u)]^2}. \end{aligned} \quad \square$$

Theorem 3.17. *Let G be a graph of order n and size m . Then*

$$\prod_1^*(PL) = \prod_1^*(G) \prod_{u \in V(G)} [2d_G(u)]^{\frac{d_G(u)(d_G(u)-1)}{2}}.$$

Proof. Since PL has $2m$ vertices, then we have

$$\prod_1^*(PL) = \prod_{uv \in E(PL)} [d_{PL}(u) + d_{PL}(v)] = \prod_1^*(G) \prod_{u \in V(G)} [2d_G(u)]^{\frac{d_G(u)(d_G(u)-1)}{2}}. \quad \square$$

Corollary 3.18. *Let G be a connected graph of order n and size m . Then*

$$\overline{\prod}_2(PL) = \frac{[\prod_2(G)]^{2m-1}}{\prod_{u \in V(G)} [d_G(u)]^{[d_G(u)]^2}}$$

Proof. From Lemma 2.2 we have

$$\overline{\prod}_2(PL) = \frac{[\prod_1(PL)]^{\frac{2m-1}{2}}}{\prod_2(PL)}.$$

From Theorems 3.15 and 3.16 we get the result. □

One can easily obtain the expressions for multiplicative Zagreb indices and coindices of line graph L of graph G by considering edge degrees of G .

Theorem 3.19. *Let G be a graph of order n and size m . Then*

- (i) $\prod_1(L) = \prod_{e \in E(G)} d_G(e)^2,$
- (ii) $\prod_2(L) = \prod_{e_i \sim e_j} d_G(e_i)d_G(e_j),$
- (iii) $\prod_1^*(L) = \prod_{e_i \sim e_j} [d_G(e_i) + d_G(e_j)],$
- (iv) $\overline{\prod}_1(L) = \prod_{e_i \not\sim e_j} [d_G(e_i) + d_G(e_j)],$
- (v) $\overline{\prod}_2(L) = \prod_{e_i \not\sim e_j} d_G(e_i)d_G(e_j),$

where $e_i \sim e_j$ (resp. $e_i \not\sim e_j$) means that the edges e_i and e_j are adjacent (resp. not adjacent) in G .

It remains a task for the future to find the expressions for $\prod_1^*(T_1)$ and $\prod_1^*(T)$.

In [19], the total multiplicative sum Zagreb index $\prod^T(G)$ of a graph G is defined as

$$\prod^T(G) = \prod_{u,v \in V(G)} [d_G(u) + d_G(v)].$$

Lemma 3.20 ([19]). *For a connected graph G , we have $\prod_1^*(G)\overline{\prod}_1(G) = \prod^T(G)$.*

By Lemma 3.20, one can find the expression for $\overline{\prod}_1$ of derived graphs. But obtaining the expression for \prod^T of derived graphs is a difficult task.

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REFERENCES

- [1] M. Azari, A. Iranmanesh, *Some inequalities for the multiplicative sum Zagreb index of graph operations*, Journal of Mathematical Inequalities **9** (2015) 3, 727–738.
- [2] B. Basavanagoud, I. Gutman, V.R. Desai, *Zagreb indices of generalized transformation graphs and their complements*, Kragujevac J. Sci. **37** (2015), 99–112.
- [3] B. Basavanagoud, I. Gutman, C.S. Gali, *On second Zagreb index and coindex of some derived graphs*, Kragujevac J. Sci. **37** (2015), 113–121.
- [4] K.C. Das, A. Yurttas, M. Togan, A.S. Cevik, I.N. Cangul, *The multiplicative Zagreb indices of graph operations*, Journal of Inequality and Applications (2013), 2013:90.
- [5] M. Eliasi, A. Iranmanesh, I. Gutman, *Multiplicative versions of first Zagreb index*, MATCH Commun. Math. Comput. Chem. **68** (2012), 217–230.
- [6] I. Gutman, *Multiplicative Zagreb indices of trees*, Bull. Internat. Math. Virt. Inst. **1** (2011), 13–19.
- [7] I. Gutman, *Degree-based topological indices*, Croat. Chem. Acta **86** (2013), 351–361.
- [8] I. Gutman, B. Furtula, Ž. Kovijanić Vukićević, G. Popivoda, *Zagreb indices and coindices*, MATCH Commun. Math. Comput. Chem. **74** (2015), 5–16.
- [9] F. Harary, *Graph Theory*, Addison-Wesley, Reading, Mass 1969.
- [10] J. Liu, Q. Zhang, *Sharp upper bounds for multiplicative Zagreb indices*, MATCH Commun. Math. Comput. Chem. **68** (2012), 231–240.
- [11] H. Narumi, M. Katayama, *Simple topological index. A newly devised index characterizing the topological nature of structural isomers of saturated hydrocarbons*, Mem. Fac. Engin. Hokkaido Univ. **16** (1984), 209–214.
- [12] T. Réti, I. Gutman, *Relations between ordinary and multiplicative Zagreb indices*, Bull. Internat. Math. Virt. Inst. **2** (2012), 133–140.
- [13] R. Todeschini, V. Consonni, *New local vertex invariants and molecular descriptors based on functions of the vertex degrees*, MATCH Commun. Math. Comput. Chem. **64** (2010), 359–372.
- [14] R. Todeschini, D. Ballabio, V. Consonni, *Novel molecular descriptors based on functions of new vertex degrees*, [in:] I. Gutman, B. Furtula (eds), *Novel molecular structure descriptors – Theory and applications I*, Univ. Kragujevac, 2010, 73–100.
- [15] Ž. Tomović, I. Gutman, *Narumi-Katayama index of phenylenes*, J. Serb. Chem. Sco. **66** (2001) 4, 243–247.
- [16] H. Wang, H. Bao, *A note on multiplicative sum Zagreb index*, South Asian J. Math. **2** (2012) 6, 578–583.
- [17] S. Wang, B. Wei, *Multiplicative Zagreb indices of k -tree*, Discrete Applied Math. **180** (2015), 168–175.
- [18] K. Xu, K.C. Das, *Trees, unicyclic and bicyclic graphs extremal with respect to multiplicative sum Zagreb index*, MATCH Commun. Math. Comput. Chem. **68** (2012), 257–272.

- [19] K. Xu, K.C. Das, K. Tang, *On the multiplicative Zagreb coindex of graphs*, *Opuscula Math.* **33** (2013) 1, 191–204.
- [20] K. Xu, H. Hua, *A unified approach to extremal multiplicative Zagreb indices for trees, unicyclic and bicyclic graphs*, *MATCH Commun. Math. Comput. Chem.* **68** (2012), 241–256.

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