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FOUR POSITIVE PERIODIC SOLUTIONS OF A DISCRETE TIME LOTKA-VOLTERRA COMPETITIVE SYSTEM WITH HARVESTING TERMS

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Abstract. In this paper, by using Mawhin's continuation theorem of coincidence degree theory, we establish the existence of at least four positive periodic solutions for a discrete time Lotka-Volterra competitive system with harvesting terms. An example is given to illustrate the effectiveness of our results.

Keywords: discrete systems, Lotka-Volterra competitive models, coincidence degree, harvesting terms.

Mathematics Subject Classification: 34A34, 39A10.

1. INTRODUCTION

Generally, the model of a two species Lotka-Volterra competitive with harvesting terms is described as [1, 2]:

$$\begin{cases} \dot{x} = x\big(a_1 - b_1x - c_1y\big) - h_1, \\ \dot{y} = y\big(a_2 - b_2y - c_2x\big) - h_2, \end{cases} \tag{1.1}$$

where x and y are functions of time representing the densities of two competitive species, respectively; $h_i, i = 1, 2$ are exploited terms of ith species standing for harvest; $a_i, b_i, c_i, i = 1, 2$ are the intrinsic growth rates, death rate, competitive rates, respectively. Moreover, we always assume that all of the parameters are positive constants.

Considering the inclusion of the effect of a changing environment, that is the following model:

$$\begin{cases} \dot{x(t)} = x(t)\big(a_1(t) - b_1(t)x(t) - c_1(t)y(t)\big) - h_1(t), \\ \dot{y(t)} = y(t)\big(a_2(t) - b_2(t)y(t) - c_2(t)x(t)\big) - h_2(t), \end{cases} \tag{1.2}$$

where $a_i(t), b_i(t), c_i(t)$ and $h_i(t), i = 1, 2$ are all positive continuous ω-periodic functions.

Since many authors [3–6] have argued that the discrete time models governed by difference equations are more appropriate than the continuous ones when the populations have non-overlapping generations, also, since discrete time models can also provide efficient computational models of continuous models for numerical simulations, it is reasonable to study a discrete time predator-prey system with harvesting terms governed by difference equations. One of the way of deriving difference equations modelling the dynamics of populations with non-overlapping generations is based on appropriate modifications of the corresponding models with overlapping generations [7,8]. In this approach, differential equations with piecewise constant arguments have proved to be useful.

Following the same idea and the same method [7, 8], one can easily derive the following discrete analogues of system (1.2), that is

$$\begin{cases} x(k+1) = x(k) \exp\left[a_1(k) - b_1(k)x(k) - c_1(k)y(k) - \dfrac{h_1(k)}{x(k)}\right], \\ y(k+1) = y(k) \exp\left[a_2(k) - b_2(k)y(k) - c_2(k)x(k) - \dfrac{h_2(k)}{y(k)}\right]. \end{cases} \tag{1.3}$$

where $a_i(k), b_i(k), c_i(k), h_i(k)$, $i = 1, 2$ are positive ω-periodic sequences, ω is a fixed positive integer denoting the common period of all the parameters in system (1.3).

In recent years, the coincidence degree has been applied to study the existence of a periodic solution or multiple periodic solutions in delayed differential population models and many good results have been obtained, see e.g. [9–15]. However, there are few papers published on multiple periodic solutions for discrete models governed by difference equations. For system (1.3), to the best of our knowledge, there is no result on multiple periodic solutions in the literature. So, in this paper, our purpose is to study the existence of four positive periodic solutions for system (1.3) by employing the continuation theorem of coincidence degree theory. Since the discrete system is more difficult to deal with. we will employ new arguments in our discussion.

The organization of this paper is as follows. In Section 2, we introduce some notations and definitions, and state some preliminary results needed in later sections. In Section 3, we shall use Mawhin's continuation theorem [16] to establish the existence of periodic solutions of (1.3). In Section 4, we will give an example to illustrate the effectiveness of our results.

2. PRELIMINARIES

In this section, we shall introduce some notations and definitions, and state some preliminary results.

Let X and Z be real normed vector spaces. Let $L : \operatorname{Dom} L \subset X \to Z$ be a linear mapping and $N : X \times [0, 1] \to Z$ be a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if dim Ker L = codim Im $L < \infty$ and

Im L is closed in Z. If L is a Fredholm mapping of index zero, then there exists continuous projectors $P : X \to X$ and $Q : Z \to Z$ such that $\operatorname{Im} P = \operatorname{Ker} L$ and $\operatorname{Ker} Q = \operatorname{Im} L = \operatorname{Im}(I - Q)$, and $X = \operatorname{Ker} L \bigoplus \operatorname{Ker} P, Z = \operatorname{Im} L \bigoplus \operatorname{Im} Q$. It follows that $L|_{\operatorname{Dom} L \cap \operatorname{Ker} P} : (I-P)X \to \operatorname{Im} L$ is invertible and its inverse is denoted by K_P. If Ω is a bounded open subset of X, the mapping N is called L-compact on $\bar{\Omega} \times [0, 1]$, if $QN(\bar{\Omega} \times [0, 1])$ is bounded and $K_P(I - Q)N : \bar{\Omega} \times [0, 1] \to X$ is compact. Because Im Q is isomorphic to Ker L, there exists an isomorphism $J : \operatorname{Im} Q \to \operatorname{Ker} L$.

The notation **deg** following means **coincidence degree** [16], the Mawhin's continuous theorem [16, p. 40] is given as follows:

Lemma 2.1 ([16]). *Let L be a Fredholm mapping of index zero and let N be L-compact on $\bar{\Omega} \times [0, 1]$. Assume:*

(a) *for each $\lambda \in (0, 1)$, every solution x of $Lx = \lambda N(x, \lambda)$ is such that $x \notin \partial\Omega \cap \operatorname{Dom} L$;*
(b) *$QN(x, 0) \neq 0$ for each $x \in \partial\Omega \cap \operatorname{Ker} L$;*
(c) $\deg(JQN(x, 0), \Omega \cap \operatorname{Ker} L, 0) \neq 0$.

Then $Lx = N(x, 1)$ has at least one solution in $\overline{\Omega} \cap \operatorname{Dom} L$.

For the sake of convenience, we introduce the following notation:

$$I_\omega = \{0, 1, \cdots, \omega - 1\}, \quad Z_0 = \{0, \pm 1, \pm 2, \cdots, \pm n, \cdots\},$$

$$g^L = \min_{k \in I_\omega} g(k), \quad g^M = \max_{k \in I_\omega} g(k), \quad \bar{g} = \frac{1}{\omega} \sum_{k=0}^{\omega-1} g(k),$$

where $g(k)$ is a ω-periodic sequence of real numbers defined for $k \in Z_0$.

Lemma 2.2 ([14]). *Let $x > 0, y > 0, z > 0$ and $x > 2\sqrt{yz}$, for the functions $f(x, y, z) = \dfrac{x + \sqrt{x^2 - 4yz}}{2z}$ and $g(x, y, z) = \dfrac{x - \sqrt{x^2 - 4yz}}{2z}$, the follow assertions hold.*

(1) *$f(x, y, z), g(x, y, z)$ are monotonically increasing and monotonically decreasing on the variable x, respectively.*
(2) *$f(x, y, z), g(x, y, z)$ are monotonically decreasing and monotonically increasing on the variable y, respectively.*
(3) *$f(x, y, z), g(x, y, z)$ are monotonically decreasing and monotonically increasing on the variable z, respectively.*

In order to apply coincidence to our study system (1.3), we will state the following definitions and propositions. For details and proof, see [15].

Define $l^\omega = \{u(k) = (u_1(k), u_2(k))^T : u_i(k + \omega) = u_i(k), k \in Z_0, i = 1, 2\}$. For $a = (a_1, a_2)^T \in R^2$, define $|a| = \max\{a_1, a_2\}$. Let $\|u(k)\| = \max_{k \in \omega} |u(k)|$, for $u(k) \in l^\omega$. Equipped with above norm $\| \cdot \|$, l_ω is a finite-dimensional Banach space.

Let

$$l_0^\omega = \left\{u(k) = \{(u_1(k), u_2(k))^T\} \in l^\omega : \sum_{k=0}^{\omega-1} u_i(k) = 0, i = 1, 2\right\},$$

$$l_c^\omega = \left\{u(k) = \{(u_1(k), u_2(k))^T\} \in l^\omega : u_i(k) = u_i \in R, i = 1, 2\right\},$$

then it follows that l_0^ω and l_c^ω are both closed linear subspaces of l^ω and

$$l^\omega = l_0^\omega \bigoplus l_c^\omega, \quad \dim l_c^\omega = 2.$$

Now we state the main result in this paper.

Throughout this paper, we assume that:

(H_1) $a_1^L - \dfrac{c_1^M a_2^M}{b_2^L} > 2\sqrt{b_1^M h_1^M}$;

(H_2) $a_2^L - \dfrac{c_2^M a_1^M}{b_1^L} > 2\sqrt{b_2^M h_2^M}$.

For convenience, we note

$$A_1^\pm = \frac{a_1^L - \dfrac{c_1^M a_2^M}{b_2^L} \pm \sqrt{[a_1^L - \dfrac{c_1^M a_2^M}{b_2^L}]^2 - 4b_1^M h_1^M}}{2b_1^M},$$

$$A_2^\pm = \frac{a_2^L - \dfrac{c_2^M a_1^M}{b_1^L} \pm \sqrt{[a_2^L - \dfrac{c_2^M a_1^M}{b_1^L}]^2 - 4b_2^M h_2^M}}{2b_2^M}.$$

3. EXISTENCE OF AT LEAST FOUR POSITIVE PERIODIC SOLUTIONS

In this section, by using Mawhin's continuation theorem, we shall show the existence of at least four positive periodic solutions of (1.3).

Theorem 3.1. *Assume that* (H_1) *and* (H_2) *hold. Then system* (1.3) *has at least four positive* ω*-periodic solutions.*

Proof. In order to use the continuation theorem of coincidence degree theory to establish the existence of solutions of (1.3), we make the substitution

$$x(k) = \exp\{u_1(k)\}, \quad y(k) = \exp\{u_2(k)\}. \tag{3.1}$$

Then system (1.3) can be reformulated as

$$\begin{cases} u_1(k+1) - u_1(k) = a_1(k) - b_1(k)e^{u_1(k)} - c_1(k)e^{u_2(k)} - \dfrac{h_1(k)}{e^{u_1(k)}}, \\ u_2(k+1) - u_2(k) = a_2(k) - b_2(k)e^{u_2(k)} - c_2(k)e^{u_1(k)} - \dfrac{h_2(k)}{e^{u_2(k)}}. \end{cases} \tag{3.2}$$

We prefer to study system (3.2) in the sequel because it is more convenient for our further discussion.

Now we define $X = Z = l^\omega, (Lu)(k) = u(k+1) - u(k)$ and

$$N(u,\lambda)(k) = \begin{pmatrix} a_1(k) - b_1(k)e^{u_1(k)} - \lambda c_1(k)e^{u_2(k)} - \dfrac{h_1(k)}{e^{u_1(k)}} \\ a_2(k) - b_2(k)e^{u_2(k)} - \lambda c_2(k)e^{u_1(k)} - \dfrac{h_2(k)}{e^{u_2(k)}} \end{pmatrix},$$

for $u \in X$ and $k \in Z_0$. It is trivial to see that L is a bounded linear operator and

$$\operatorname{Ker} L = l_c^\omega, \quad \operatorname{Im} L = l_0^\omega$$

as well as

$$\dim \operatorname{Ker} L = 2 = \operatorname{codim} \operatorname{Im} L;$$

then it follows that L is a Fredholm mapping of index zero. Define

$$Pu = \frac{1}{\omega}\sum_{k=0}^{\omega-1} u(k), u \in X, \qquad Qz = \frac{1}{\omega}\sum_{k=0}^{\omega-1} z(k), z \in Z.$$

It is easy to show that P and Q are continuous projections such that

$$\operatorname{Im} P = \operatorname{Ker} L, \qquad \operatorname{Im} L = \operatorname{Ker} Q = \operatorname{Im}(I-Q).$$

Moreover, the generalized inverse (to L) $K_p : \operatorname{Im} L \to \operatorname{Ker} P \cap \operatorname{Dom} L$ exists and is given by

$$K_p(z) = \sum_{s=0}^{\omega-1} z(s) - \frac{1}{\omega}\sum_{s=0}^{\omega-1}(\omega - s)z(s).$$

Clearly, QN and $K_P(I-Q)N$ are continuous. Since X is a finite-dimensional Banach space, one can easily show that $K_P(I-Q)N(\bar{\Omega}\times[0,1])$ are relatively compact for any open bounded set $\Omega \subset X$. $QN(\bar{\Omega}\times[0,1])$ is bounded, and hence N is L-compact on $\bar{\Omega}$ for any open bounded set $\Omega \subset X$. Now we reach the position to search for an appropriate open, bounded subset Ω for the application of the continuation theorem.

In order to use Lemma 2.1, we have to find at least four appropriate open bounded subsets in X. Considering the operator equation$Lu = \lambda N(u,\lambda)$, that is

$$\begin{cases} u_1(k+1) - u_1(k) = \lambda\left[a_1(k) - b_1(k)e^{u_1(k)} - \lambda c_1(k)e^{u_2(k)} - \dfrac{h_1(k)}{e^{u_1(k)}}\right], \\ u_2(k+1) - u_2(k) = \lambda\left[a_2(k) - b_2(k)e^{u_2(k)} - \lambda c_2(k)e^{u_1(k)} - \dfrac{h_2(k)}{e^{u_2(k)}}\right]. \end{cases} \tag{3.3}$$

Summing (3.3) from 0 to $\omega - 1$ gives

$$\begin{cases} 0 = \lambda \sum\limits_{k=0}^{\omega-1}\left[a_1(k) - b_1(k)e^{u_1(k)} - \lambda c_1(k)e^{u_2(k)}\dfrac{h_1(k)}{e^{u_1(k)}}\right], \\ 0 = \lambda \sum\limits_{k=0}^{\omega-1}\left[a_2(k) - b_2(k)e^{u_2(k)} - \lambda c_2(k)e^{u_1(k)}\dfrac{h_2(k)}{e^{u_2(k)}}\right]. \end{cases} \tag{3.4}$$

From the above two equations, we get

$$\sum_{k=0}^{\omega-1} a_1(k) = \sum_{k=0}^{\omega-1} \left[b_1(k)e^{u_1(k)} + \lambda c_1(k)e^{u_2(k)} + \frac{h(_1k)}{e^{u_1(k)}} \right] \tag{3.5}$$

and

$$\sum_{k=0}^{\omega-1} a_2(k) = \sum_{k=0}^{\omega-1} \left[b_2(k)e^{u_2(k)} + \lambda c_2(k)e^{u_1(k)} + \frac{h_2(k)}{e^{u_2(k)}} \right]. \tag{3.6}$$

From (3.4)–(3.6), we get

$$\sum_{k=0}^{\omega-1} |u_1(k+1) - u_1(k)| \leq \sum_{k=0}^{\omega-1} \left[a_1(k) + b_1(k)e^{u_1(k)} + \lambda c_1(k)e^{u_2(k)} + \frac{h_1(k)}{e^{u_1(k)}} \right] = 2\bar{a}_1\omega \tag{3.7}$$

and

$$\sum_{k=0}^{\omega-1} |u_2(k+1) - u_2(k)| \leq \sum_{k=0}^{\omega-1} \left[a_2(k) + b_2(k)e^{u_2(k)} + \lambda c_2(k)e^{u_1(k)} + \frac{h_2(k)}{e^{u_2(k)}} \right] = 2\bar{a}_2\omega. \tag{3.8}$$

Assume that $u = (u_1, u_2)^T \in X$ is an ω-periodic solution of system (3.3) for some $\lambda \in (0, 1)$. Then there exists $\xi_i, \eta_i \in I_\omega$ such that

$$u_i(\xi_i) = \max_{k \in I_\omega} u_i(k), \quad u_i(\eta_i) = \min_{k \in I_\omega} u_i(k), i = 1, 2. \tag{3.9}$$

From this and (3.4), we have

$$u_1(\eta_1+1) - u_1(\eta_1) = \lambda \left[a_1(\eta_1) - b_1(\eta_1)e^{u_1(\eta_1)} - \lambda c_1(\eta_1)e^{u_2(\eta_1)} - \frac{h_1(\eta_1)}{e^{u_1(\eta_1)}} \right] \geq 0 \tag{3.10}$$

and

$$u_2(\eta_2+1) - u_2(\eta_2) = \lambda \left[a_2(\eta_2) - b_2(\eta_2)e^{u_2(\eta_2)} - \lambda c_2(\eta_2)e^{u_1(\eta_2)} - \frac{h_2(\eta_2)}{e^{u_2(\eta_2)}} \right] \geq 0. \tag{3.11}$$

From this, we have

$$b_1^L e^{2u_1(\eta_1)} - a_1^M e^{u_1(\eta_1)} + h_1^L < 0$$

and

$$b_2^L e^{2u_2(\eta_2)} - a_2^M e^{u_2(\eta_2)} + h_2^L < 0,$$

which imply that

$$\ln l_i^- < u_i(\eta_i) < \ln l_i^+, \quad i = 1, 2. \tag{3.12}$$

From (3.4), we have

$$b_1^L e^{u_1(\xi_1)} < b_1(\xi_1)e^{u_1(\xi_1)} + h_1(\xi_1)e^{-u_1(\xi_1)} = a_1(\xi_1) - \lambda c_1(\xi_1)e^{u_2(\xi_1)} < a_1(\xi_1) \leq a_1^M$$

and

$$b_2^L e^{u_2(\xi_2)} < b_2(\xi_2)e^{u_2(\xi_2)} + h_2(\xi_2)e^{-u_2(\xi_2)} = a_2(\xi_2) - \lambda c_2(\xi_2)e^{u_1(\xi_2)} < a_2(\xi_2) \le a_2^M,$$

which imply that

$$u_1(\eta_1) \le u_1(\xi_1) < \ln \frac{a_1^M}{b_1^L} \tag{3.13}$$

and

$$u_2(\eta_2) \le u_2(\xi_2) < \ln \frac{a_2^M}{b_2^L}. \tag{3.14}$$

From (3.4) and (3.9), we have

$$\bar{a_1}\omega > \sum_{k=0}^{\omega-1} \frac{h_1(k)}{e^{u_1(k)}} \ge \frac{\bar{h_1}\omega}{e^{u_1(\xi_1)}}$$

and

$$\bar{a_2}\omega > \sum_{k=0}^{\omega-1} \frac{h_2(k)}{e^{u_2(k)}} \ge \frac{\bar{h_2}\omega}{e^{u_2(\xi_2)}},$$

that is

$$u_1(\xi_1) > \ln \frac{\bar{h_1}}{\bar{a_1}} \ge \ln \frac{h_1^L}{a_1^M}$$

and

$$u_2(\xi_2) > \ln \frac{\bar{h_2}}{\bar{a_2}} \ge \ln \frac{h_2^L}{a_2^M}.$$

For $k \in I_\omega$, from this and (3.7), (3.8), (3.13) and (3.14) gives

$$u_i(k) \ge u_i(\xi_i) - \sum_{k=0}^{\omega-i} |u_i(k+1) - u_i(k)| > \ln \frac{h_i^L}{a_i^M} - 2\bar{a}_i\omega := H_i^1, \quad i = 1, 2, \tag{3.15}$$

and

$$u_i(k) \le u_i(\eta_i) + \sum_{k=0}^{\omega-1} |u_i(k+1) - u_i(k)| < \ln \frac{a_i^M}{b_i^L} + 2\bar{a}_i\omega := H_i^2, \quad i = 1, 2. \tag{3.16}$$

From (3.4) and (3.9), we have

$$u_1(\xi_1+1) - u_1(\xi_1) = \lambda\left[a_1(\xi_1) - b_1(\xi_1)e^{u_1(\xi_1)} - \lambda c_1(\xi_1)e^{u_2(\xi_1)} - \frac{h_1(\xi_1)}{e^{u_1(\xi_1)}}\right] \le 0$$

and

$$u_2(\xi_2+1) - u_2(\xi_2) = \lambda\left[a_2(\xi_2) - b_2(\xi_2)e^{u_2(\xi_2)} - \lambda c_2(\xi_2)e^{u_1(\xi_2)} - \frac{h_2(\xi_2)}{e^{u_2(\xi_2)}}\right] \le 0.$$

From this, we have

$$b_1^M e^{2u_1(\xi_1)} - \left[a_1^L - \frac{c_1^M a_2^M}{b_2^L}\right] e^{u_1(\xi_1)} + h_1^M > 0$$

and

$$b_2^M e^{2u_2(\xi_2)} - \left[a_2^L - \frac{c_2^M a_1^M}{b_1^L}\right] e^{u_2(\xi_2)} + h_2^M > 0,$$

which imply that

$$u_1(\xi_1) < \ln A_1^- \quad \text{or} \quad u_1(\xi_1) > \ln A_1^+ \tag{3.17}$$

and

$$u_2(\xi_2) < \ln A_2^- \quad \text{or} \quad u_2(\xi_2) > \ln A_2^+. \tag{3.18}$$

By the Lemma 2.2, it is easy to verify that

$$H_i^1 < \ln l_i^- < \ln A_i^- < \ln A_i^+ < \ln l_i^+ < H_i^2, \quad i = 1, 2. \tag{3.19}$$

Where

$$l_i^{\pm} = \frac{a_i^M \pm \sqrt{(a_i^M)^2 - 4b_i^L h_i^L}}{2b_i^L}, \quad i = 1, 2.$$

Clearly, $A_i^{\pm}, H_i^1, H_i^2, l_i^{\pm}, i = 1, 2.$ are independent of λ.

Now let us consider $QN(u, 0)$ with $u = (u_1, u_2)^T \in R^2$. Note that

$$QN(u_1, u_2; 0) = \begin{pmatrix} \bar{a_1} - \bar{b_1} e^{u_1} - \dfrac{\bar{h_1}}{e^{u_1}} \\ \\ \bar{a_2} - \bar{b_2} e^{u_2} - \dfrac{\bar{h_2}}{e^{u_2}} \end{pmatrix}.$$

Since H_1 and H_2 hold, then $\bar{a_i} > 2\sqrt{\bar{b_i}\bar{h_i}}, i = 1, 2$, we can show that $QN(u_1, u_2; 0)$ has four distinct solutions:

$$(u_1^1, u_2^1) = (\ln x_-, \ln y_-), \qquad (u_1^2, u_2^2) = (\ln x_-, \ln y_+),$$

$$(u_1^3, u_2^3) = (\ln x_+, \ln y_-), \qquad (u_1^4, u_2^4) = (\ln x_+, \ln y_+),$$

where

$$x_{\pm} = \frac{\bar{a_1} \pm \sqrt{(\bar{a_1})^2 - 4\bar{b_1}\bar{h_1}}}{2\bar{b_1}}, \qquad y_{\pm} = \frac{\bar{a_2} \pm \sqrt{(\bar{a_2})^2 - 4\bar{b_2}\bar{h_2}}}{2\bar{b_2}}.$$

It is easy to verify that

$$H_1^1 < \ln l_1^- < \ln x_- < \ln A_1^- < \ln A_1^+ < \ln x_+ < \ln l_1^+ < H_1^2$$

and

$$H_2^1 < \ln l_2^- < \ln y_- < \ln A_2^- < \ln A_2^+ < \ln y_+ < \ln l_2^+ < H_2^2.$$

Let

$$\Omega_1 = \left\{ u = (u_1, u_2)^T \in X \middle| \begin{array}{c} u_1(k) \in (\ln l_1^-, \ln A_1^-) \\ u_2(k) \in (\ln l_2^-, \ln A_2^-) \end{array} \right\},$$

$$\Omega_2 = \left\{ u = (u_1, u_2)^T \in X \middle| \begin{array}{c} u_1(k) \in ((\ln l_1^-, \ln A_1^-) \\ \min\limits_{k \in I_\omega} u_2(k) \in (\ln l_2^-, \ln l_2^+) \\ \max\limits_{k \in I_\omega} u_2(k) \in (\ln A_2^+, \ln H_2^2) \end{array} \right\},$$

$$\Omega_3 = \left\{ u = (u_1, u_2)^T \in X \middle| \begin{array}{c} \min\limits_{k \in I_\omega} u_1(k) \in (\ln l_1^-, \ln l_1^+) \\ \max\limits_{k \in I_\omega} u_1(k) \in (\ln A_1^+, \ln H_1^2) \\ u_2(k) \in (\ln l_2^-, \ln A_2^-) \end{array} \right\},$$

$$\Omega_4 = \left\{ u = (u_1, u_2)^T \in X \middle| \begin{array}{c} \min\limits_{k \in I_\omega} u_1(k) \in (\ln l_1^-, \ln l_1^+) \\ \max\limits_{k \in I_\omega} u_1(k) \in (\ln A_1^+, \ln H_1^2) \\ \min\limits_{k \in I_\omega} u_2(k) \in (\ln l_2^-, \ln l_2^+) \\ \max\limits_{k \in I_\omega} u_2(k) \in (\ln A_2^+, \ln H_2^2) \end{array} \right\}.$$

It is easy to see that $(u_1^i, u_2^i) \in \Omega_i, i = 1, 2, 3, 4$ and Ω_i are open bounded subset of X. With the help of (3.10)-(3.20), it is not difficult to verify that $\Omega_i \cap \Omega_j = \emptyset, i \neq j$ and Ω_i satisfies condition (a) of Lemma 2.1 Moreover, when $u \in \partial\Omega_i \cap \operatorname{Ker} L$, $i = 1, 2, 3, 4, QN(u, 0) \neq (0, 0)^T$, so condition (b) of Lemma 2.1 holds.

Finally, we shall show that condition (c) of Lemma 2.1 holds. Since $\operatorname{Ker} L = \operatorname{Im} Q$, we can take $J = I$. A direct computation gives, for $i = 1, 2, 3, 4$,

$$\begin{aligned}
&\deg\left\{JQN(u, 0), \Omega_i \cap \text{Ker } L, (0, 0)^T\right\} = \\
&= \deg\left\{\left(\bar{a_1} - \bar{b_1}e^{u_1} - \frac{\bar{h_1}}{e^{u_1}}, \bar{a_2} - \bar{b_2}e^{u_2} - \frac{\bar{h_2}}{e^{u_2}}\right)^T, \Omega_i \cap \text{Ker } L, (0, 0)^T\right\} = \\
&= \operatorname{sign}\begin{vmatrix} -\bar{b_1}e^{u_1^i} + \bar{h_1}e^{-u_1^i} & 0 \\ 0 & -\bar{b_2}e^{u_2^i} + \bar{h_2}e^{-u_2^i} \end{vmatrix} = \\
&= \operatorname{sign}\left[(\bar{a_1} - 2\bar{b_1}e^{u_1^i})(\bar{a_2} - 2\bar{b_2}e^{u_2^i})\right] = \\
&= \pm 1 \neq 0.
\end{aligned}$$

So far, we have proved that Ω_i $(i = 1, 2, 3, 4)$ satisfies all the assumptions in Lemma 2.1. Hence, system (3.2) has at least four different ω-periodic solutions. Thus system (1.3) has at least four different positive ω-periodic solutions. This completes the proof of Theorem 3.1. □

4. AN EXAMPLE

$$\begin{cases} x(k+1) = x(k)\exp\left[3+\cos\frac{k\pi}{3} - \frac{4+\cos\frac{k\pi}{3}}{10}x(k) - \frac{2+\sin\frac{k\pi}{3}}{100}y(k) - \frac{9+\sin\frac{k\pi}{3}}{20x(k)}\right], \\ y(k+1) = y(k)\exp\left[3+\sin\frac{k\pi}{3} - \frac{5+\sin\frac{k\pi}{3}}{10}y(k) - \frac{2+\cos\frac{k\pi}{3}}{10}x(k) - \frac{2+\sin\frac{k\pi}{3}}{5y(k)}\right]. \end{cases} \tag{4.1}$$

Since, in this case

$$a_1^L - c_1^M \frac{a_2^M}{b_2^L} = 2 - \frac{3}{100} \times \frac{4\times 10}{4} = \frac{17}{10} > 1 = 2\sqrt{b_1^M h_1^M},$$

$$a_2^L - c_2^M \frac{a_1^M}{b_1^L} = 2 - \frac{3}{100} \times \frac{4\times 10}{3} = \frac{7}{5} > \frac{6}{5} = 2\sqrt{b_2^M h_2^M}.$$

Therefore, all conditions of Theorem 3.1 are satisfied. By Theorem 3.1, system (4.1) has at least four positive 6-periodic solutions.

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